Two-Dimensional Steady Heat Conduction

5-43C For a medium in which the finite difference formulation of a general interior node is given in its simplest form as $T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \dfrac{\dot{g}_{\text{node}} l^2}{k} = 0$:

(*a*) Heat transfer is steady, (*b*) heat transfer is two-dimensional, (*c*) there is heat generation in the medium, (*d*) the nodal spacing is constant, and (*e*) the thermal conductivity of the medium is constant.

5-44C For a medium in which the finite difference formulation of a general interior node is given in its simplest form as $T_{\text{node}} = (T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}})/4$:

(*a*) Heat transfer is steady, (*b*) heat transfer is two-dimensional, (*c*) there is no heat generation in the medium, (*d*) the nodal spacing is constant, and (*e*) the thermal conductivity of the medium is constant.

5-45C A region that cannot be filled with simple volume elements such as strips for a plane wall, and rectangular elements for two-dimensional conduction is said to have *irregular boundaries*. A practical way of dealing with such geometries in the finite difference method is to replace the elements bordering the irregular geometry by a series of simple volume elements.

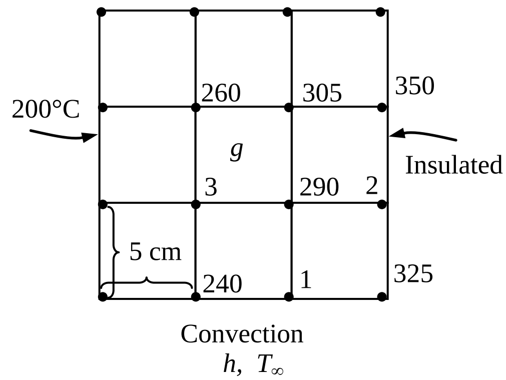
5-46 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures and the rate of heat loss from the bottom surface through a 1-m long section are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Heat is generated uniformly in the body. **3** Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be k = 45 W/m·°C.

Analysis The nodal spacing is given to be Δx=Δx=l=0.05 m, and the general finite difference form of an interior node for steady two-dimensional heat conduction is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0$$



where

$$\frac{\dot{g}_{\text{node}} l^2}{k} = \frac{\dot{g}_0 l^2}{k} = \frac{(8\times10^6 \text{ W/m}^3)(0.05\,\text{m})^2}{214\,\text{W/m}\cdot°\text{C}} = 93.5°\text{C}$$

The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}: \quad k\frac{l}{2}\frac{240-T_1}{l} + kl\frac{290-T_1}{l} + k\frac{l}{2}\frac{325-T_1}{l} + hl(T_\infty - T_1) + \frac{\dot{g}_0 l^2}{2k} = 0$$

$$\text{Node 2 (interior)}: \quad 350+290+325+290-4T_2 + \frac{\dot{g}_0 l^2}{k} = 0$$

$$\text{Node 3 (interior)}: \quad 260+290+240+200-4T_3 + \frac{\dot{g}_0 l^2}{k} = 0$$

where $k = 45\,\text{W/m.°C}$, $h = 50\,\text{W/m}^2.°\text{C}$, $\dot{g} = 8\times10^6 \text{ W/m}^3$, $T_\infty = 20°\text{C}$

Substituting, T_1 **= 280.9°C**, T_2 **= 397.1°C**, T_3 **= 330.8°C**,

(b) The rate of heat loss from the bottom surface through a 1-m long section is

$$\begin{aligned}\dot{Q} &= \sum_m \dot{Q}_{\text{element},m} = \sum_m hA_{\text{surface},m}(T_m - T_\infty) \\ &= h(l/2)(200-T_\infty) + hl(240-T_\infty) + hl(T_1 - T_\infty) + h(l/2)(325-T_\infty) \\ &= (50\,\text{W/m}^2\cdot°\text{C})(0.05\,\text{m}\times1\,\text{m})[(200-20)/2+(240-20)+(280.9-20)+(325-20)/2]°\text{C} = \mathbf{1808\ W}\end{aligned}$$

5-47 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures are to be determined.

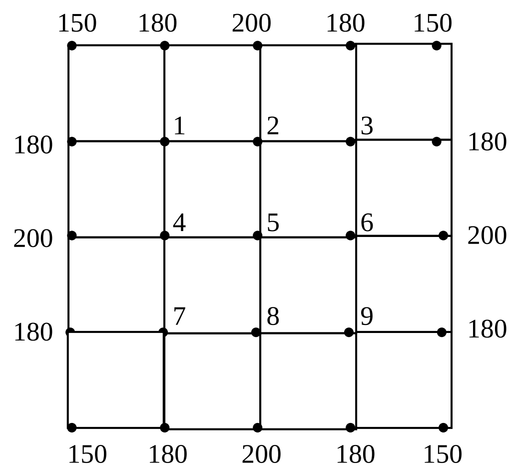
Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** There is no heat generation in the body.

Properties The thermal conductivity is given to be k = 45 W/m·°C.

Analysis The nodal spacing is given to be Δx=Δx=l=0.01 m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of no heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0 \;\rightarrow\; T_{\text{node}} = (T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}})/4$$

There is symmetry about the horizontal, vertical, and diagonal lines passing through the midpoint, and thus we need to consider only 1/8$^{\text{th}}$ of the region. Then,



$$T_1 = T_3 = T_7 = T_9$$
$$T_2 = T_4 = T_6 = T_8$$

Therefore, there are there are only 3 unknown nodal temperatures, $T_1, T_3,$ and T_5, and thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.

Node 1 (interior): $T_1 = (180 + 180 + 2T_2)/4$

Node 2 (interior): $T_2 = (200 + T_5 + 2T_1)/4$

Node 3 (interior): $T_5 = 4T_2/4 = T_2$

Solving the equations above simultaneously gives

$$T_1 = T_3 = T_7 = T_9 = \mathbf{185°C}$$
$$T_2 = T_4 = T_5 = T_6 = T_8 = \mathbf{190°C}$$

Discussion Note that taking advantage of symmetry simplified the problem greatly.

5-48 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures are to be determined.

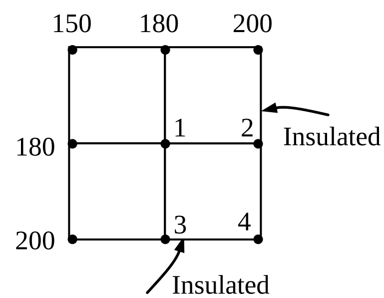
Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** There is no heat generation in the body.

Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis The nodal spacing is given to be Δx=Δx=l=0.02 m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of no heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0 \;\rightarrow\; T_{\text{node}} = (T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}})/4$$

(a) There is symmetry about the insulated surfaces as well as about the diagonal line. Therefore $T_3 = T_2$, and T_1, T_2, and T_4 are the only 3 unknown nodal temperatures. Thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.



Node 1 (interior): $T_1 = (180 + 180 + T_2 + T_3)/4$

Node 2 (interior): $T_2 = (200 + T_4 + 2T_1)/4$

Node 4 (interior): $T_4 = (2T_2 + 2T_3)/4$

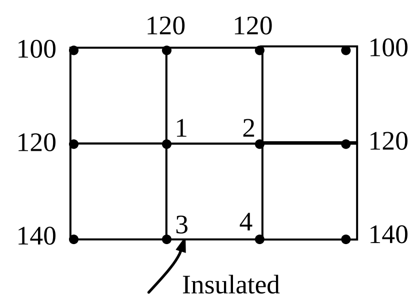
Also, $T_3 = T_2$

Solving the equations above simultaneously gives

$T_2 = T_3 = T_4 = \mathbf{190°C}$

$T_1 = \mathbf{185°C}$

(b) There is symmetry about the insulated surface as well as the diagonal line. Replacing the symmetry lines by insulation, and utilizing the mirror-image concept, the finite difference equations for the interior nodes can be written as



Node 1 (interior): $T_1 = (120 + 120 + T_2 + T_3)/4$

Node 2 (interior): $T_2 = (120 + 120 + T_4 + T_1)/4$

Node 3 (interior): $T_3 = (140 + 2T1 + T_4)/4 = T_2$

Node 4 (interior): $T_4 = (2T_2 + 140 + 2T_3)/4$

Solving the equations above simultaneously gives

$T_1 = T_2 = \mathbf{122.9°C}$

$T_3 = T_4 = \mathbf{128.6°C}$

Discussion Note that taking advantage of symmetry simplified the problem greatly.

5-49 Starting with an energy balance on a volume element, the steady two-dimensional finite difference equation for a general interior node in rectangular coordinates for $T(x, y)$ for the case of variable thermal conductivity and uniform heat generation is to be obtained.

Analysis We consider a *volume element* of size $\Delta x \times \Delta y \times 1$ centered about a general interior node (m, n) in a region in which heat is generated at a constant rate of $\dot{g}$ and the thermal conductivity k is variable (see Fig. 5-24 in the text). Assuming the direction of heat conduction to be *towards* the node under consideration at all surfaces, the energy balance on the volume element can be expressed as

$$\dot{Q}_{\text{cond, left}} + \dot{Q}_{\text{cond, top}} + \dot{Q}_{\text{cond, right}} + \dot{Q}_{\text{cond, bottom}} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} = 0$$

for the *steady* case. Again assuming the temperatures between the adjacent nodes to vary linearly and noting that the heat transfer area is $\Delta y \times 1$ in the x direction and $\Delta x \times 1$ in the y direction, the energy balance relation above becomes

$$k_{m,n}(\Delta y \times 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k_{m,n}(\Delta x \times 1)\frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k_{m,n}(\Delta y \times 1)\frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$+ k_{m,n}(\Delta x \times 1)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + \dot{g}_0(\Delta x \times \Delta y \times 1) = 0$$

Dividing each term by $\Delta x \times \Delta y \times 1$ and simplifying gives

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} + \frac{\dot{g}_0}{k_{m,n}} = 0$$

For a square mesh with $\Delta x = \Delta y = l$, and the relation above simplifies to

$$T_{m-1,n} + T_{m+1,n} + T_{m,n-1} + T_{m,n-1} - 4T_{m,n} + \frac{\dot{g}_0 l^2}{k_{m,n}} = 0$$

It can also be expressed in the following easy-to-remember form:

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_0 l^2}{k_{\text{node}}} = 0$$

5-50 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures and the rate of heat loss from the top surface are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Heat is generated uniformly in the body.

Properties The thermal conductivity is given to be k = 180 W/m·°C.

Analysis (*a*) The nodal spacing is given to be $\Delta x=\Delta x=l$=0.1 m, and the general finite difference form of an interior node equation for steady two-dimensional heat conduction for the case of constant heat generation is expressed as

$$T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}+\frac{\dot{g}_{\text{node}}l^2}{k}=0$$

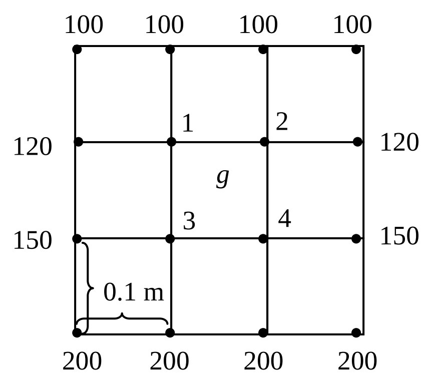
There is symmetry about a vertical line passing through the middle of the region, and thus we need to consider only half of the region. Then,

$$T_1=T_2 \text{ and } T_3=T_4$$

Therefore, there are there are only 2 unknown nodal temperatures, T_1 and T_3, and thus we need only 2 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.

Node 1 (interior): $\quad 100+120+T_2+T_3-4T_1+\frac{\dot{g}l^2}{k}=0$

Node 3 (interior): $\quad 150+200+T_1+T_4-4T_3+\frac{\dot{g}l^2}{k}=0$



Noting that $T_1=T_2$ and $T_3=T_4$ and substituting,

$$220+T_3-3T_1+\frac{(10^7\ \text{W/m}^3)(0.1\text{m})^2}{180\,\text{W/m}\cdot{}^\circ\text{C}}=0$$

$$350+T_1-3T_3+\frac{(10^7\ \text{W/m}^3)(0.1\text{m})^2}{180\,\text{W/m}\cdot{}^\circ\text{C}}=0$$

The solution of the above system is

$$T_1=T_2=\mathbf{411.5^\circ C}$$
$$T_3=T_4=\mathbf{439.0^\circ C}$$

(*b*) The total rate of heat transfer from the top surface $\dot{Q}_{\text{top}}$ can be determined from an energy balance on a volume element at the top surface whose height is l/2, length 0.3 m, and depth 1 m:

$$\dot{Q}_{\text{top}}+\dot{g}_0(0.3\times1\times l/2)+\left(2k\frac{l\times1}{2}\frac{120-100}{l}+2kl\times1\frac{T_1-100}{l}\right)=0$$

$$\dot{Q}_{\text{top}}=-(10^7\ \text{W/m}^3)(0.3\times0.1/2)\text{m}^3-2(180\,\text{W/m}\cdot{}^\circ\text{C})\left(\frac{1\,\text{m}}{2}(120-100)^\circ\text{C}+(1\,\text{m})(411.5\text{-}100)^\circ\text{C}\right)$$

$$=\mathbf{265{,}750\,W}\qquad(\text{per } m \text{ depth})$$

5-51 "!PROBLEM 5-51"

"GIVEN"
k=180 "[W/m-C], parameter to be varied"
g_dot=1E7 "[W/m^3], parameter to be varied"
DELTAx=0.10 "[m]"
DELTAy=0.10 "[m]"
d=1 "[m], depth"
"Temperatures at the selected nodes are also given in the figure"

"ANALYSIS"
"(a)"
l=DELTAx
T_1=T_2 "due to symmetry"
T_3=T_4 "due to symmetry"
"Using the finite difference method, the two equations for the two unknown temperatures are determined to be"
120+120+T_2+T_3-4*T_1+(g_dot*l^2)/k=0
150+200+T_1+T_4-4*T_3+(g_dot*l^2)/k=0
"(b)"
"The rate of heat loss from the top surface can be determined from an energy balance on a volume element whose height is l/2, length 3*l, and depth d=1 m"
-Q_dot_top+g_dot*(3*l*d*l/2)+2*(k*(l*d)/2*(120-100)/l+k*l*d*(T_1-100)/l)=0

k [W/m.C]	T_1 [C]	T_3 [C]	Q_{top} [W]
10	5134	5161	250875
30.53	1772	1799	252671
51.05	1113	1141	254467
71.58	832.3	859.8	256263
92.11	676.6	704.1	258059
112.6	577.7	605.2	259855
133.2	509.2	536.7	261651
153.7	459.1	486.6	263447
174.2	420.8	448.3	265243
194.7	390.5	418	267039
215.3	366	393.5	268836
235.8	345.8	373.3	270632
256.3	328.8	356.3	272428
276.8	314.4	341.9	274224
297.4	301.9	329.4	276020
317.9	291	318.5	277816
338.4	281.5	309	279612
358.9	273	300.5	281408
379.5	265.5	293	283204
400	258.8	286.3	285000

g [W/m^3]	**T_1 [C]**	**T_3 [C]**	**Q_{top} [W]**
100000	136.5	164	18250
5.358E+06	282.6	310.1	149697
1.061E+07	428.6	456.1	281145
1.587E+07	574.7	602.2	412592
2.113E+07	720.7	748.2	544039
2.639E+07	866.8	894.3	675487
3.165E+07	1013	1040	806934
3.691E+07	1159	1186	938382
4.216E+07	1305	1332	1.070E+06
4.742E+07	1451	1479	1.201E+06
5.268E+07	1597	1625	1.333E+06
5.794E+07	1743	1771	1.464E+06
6.319E+07	1889	1917	1.596E+06
6.845E+07	2035	2063	1.727E+06
7.371E+07	2181	2209	1.859E+06
7.897E+07	2327	2355	1.990E+06
8.423E+07	2473	2501	2.121E+06
8.948E+07	2619	2647	2.253E+06
9.474E+07	2765	2793	2.384E+06
1.000E+08	2912	2939	2.516E+06

5-52 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** There is no heat generation in the body.

Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis The nodal spacing is given to be Δx=Δx=l=0.01m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of no heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0 \;\rightarrow\; T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0$$

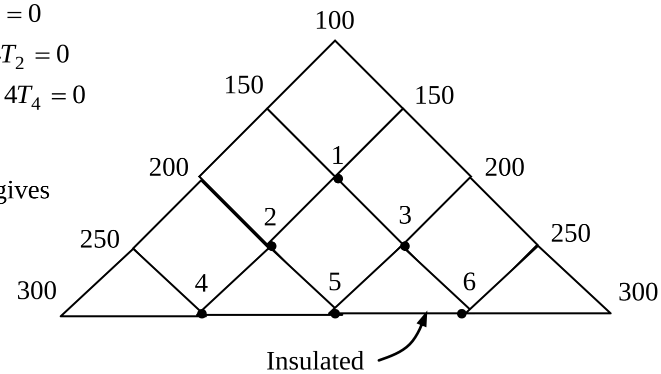
(*a*) There is symmetry about a vertical line passing through the nodes 1 and 3. Therefore, $T_3 = T_2$, $T_6 = T_4$, and $T_1, T_2, T_4,$ and T_5 are the only 4 unknown nodal temperatures, and thus we need only 4 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.

Node 1 (interior): $150 + 150 + 2T_2 - 4T_1 = 0$

Node 2 (interior): $200 + T_1 + T_5 + T_4 - 4T_2 = 0$

Node 4 (interior): $250 + 250 + T_2 + T_5 - 4T_4 = 0$

Node 5 (interior): $4T_2 - 4T_5 = 0$



Solving the 4 equations above simultaneously gives

$T_1 = \mathbf{175°C}$

$T_2 = T_3 = \mathbf{200°C}$

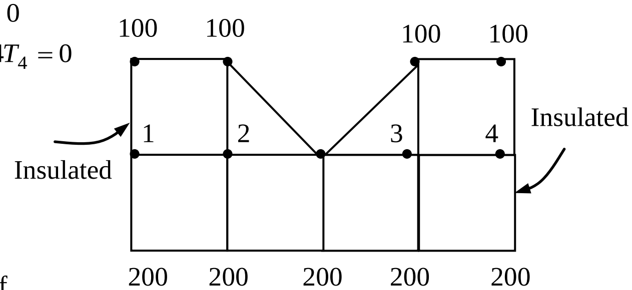
$T_4 = T_6 = \mathbf{225°C}$

$T_5 = \mathbf{200°C}$

(*b*) There is symmetry about a vertical line passing through the middle. Therefore, $T_3 = T_2$ and $T_4 = T_1$. Replacing the symmetry lines by insulation and utilizing the mirror-image concept, the finite difference equations for the interior nodes 1 and 2 are determined to be

Node 1 (interior): $100 + 200 + 2T_2 - 4T_1 = 0$

Node 2 (interior): $100 + 100 + 200 + T_1 - 4T_4 = 0$



Solving the 2 equations above simultaneously gives

$T_1 = T_4 = \mathbf{143°C}$

$T_2 = T_3 = \mathbf{136°C}$

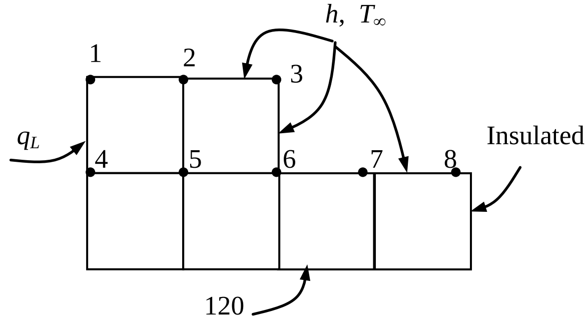
Discussion Note that taking advantage of symmetry simplified the problem greatly.

5-53 Heat conduction through a long L-shaped solid bar with specified boundary conditions is considered. The unknown nodal temperatures are to be determined with the finite difference method. √

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Thermal conductivity is constant. **3** Heat generation is uniform.

Properties The thermal conductivity is given to be k = 45 W/m·°C.

Analysis (*a*) The nodal spacing is given to be $\Delta x=\Delta x=l$=0.015 m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of constant heat generation is expressed as



$$T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}+\frac{\dot{g}_0 l^2}{k}=0$$

We observe that all nodes are boundary nodes except node 5 that is an interior node. Therefore, we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 8 nodes are obtained as follows:

Node 1: $\dot{q}_L \frac{l}{2}+h\frac{l}{2}(T_\infty-T_1)+k\frac{l}{2}\frac{T_2-T_1}{l}+k\frac{l}{2}\frac{T_4-T_1}{l}+\dot{g}_0\frac{l^2}{4}=0$

Node 2: $hl(T_\infty-T_2)+k\frac{l}{2}\frac{T_1-T_2}{l}+k\frac{l}{2}\frac{T_3-T_2}{l}+kl\frac{T_5-T_2}{l}+\dot{g}_0\frac{l^2}{2}=0$

Node 3: $hl(T_\infty-T_3)+k\frac{l}{2}\frac{T_2-T_3}{l}+k\frac{l}{2}\frac{T_6-T_3}{l}+\dot{g}_0\frac{l^2}{4}=0$

Node 4: $\dot{q}_L l+k\frac{l}{2}\frac{T_1-T_4}{l}+k\frac{l}{2}\frac{120-T_4}{l}+kl\frac{T_5-T_4}{l}+\dot{g}_0\frac{l^2}{2}=0$

Node 5: $T_4+T_2+T_6+120-4T_5+\frac{\dot{g}_0 l^2}{k}=0$

Node 6: $hl(T_\infty-T_6)+k\frac{l}{2}\frac{T_3-T_6}{l}+kl\frac{T_5-T_6}{l}+kl\frac{120-T_6}{l}+k\frac{l}{2}\frac{T_7-T_6}{l}+\dot{g}_0\frac{3l^2}{4}=0$

Node 7: $hl(T_\infty-T_7)+k\frac{l}{2}\frac{T_6-T_7}{l}+k\frac{l}{2}\frac{T_8-T_7}{l}+kl\frac{120-T_7}{l}+\dot{g}_0\frac{l^2}{2}=0$

Node 8: $h\frac{l}{2}(T_\infty-T_8)+k\frac{l}{2}\frac{T_7-T_8}{l}+k\frac{l}{2}\frac{120-T_8}{l}+\dot{g}_0\frac{l^2}{4}=0$

where $\dot{g}_0=5\times10^6$ W/m^3, $\dot{q}_L=8000\,\text{W/m}^2$, l = 0.015 m, k = 45 W/m·°C, h = 55 W/m^2·°C, and T_∞ =30°C. This system of 8 equations with 8 unknowns is the finite difference formulation of the problem.

(*b*) The 8 nodal temperatures under steady conditions are determined by solving the 8 equations above simultaneously with an equation solver to be

T_1 =163.6°C, T_2 =160.5°C, T_3 =156.4°C, T_4 =154.0°C, T_5 =151.0°C, T_6 =144.4°C, T_7 =134.5°C, T_8 =132.6°C

Discussion The accuracy of the solution can be improved by using more nodal points.

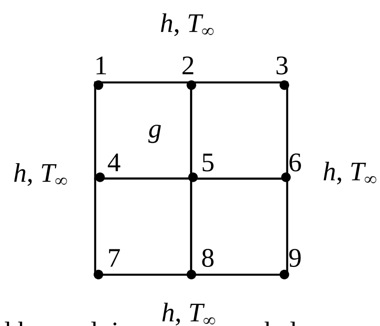
5-54E A long solid bar is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures and the rate of heat loss from the bar through a 1-ft long section are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Heat is generated uniformly in the body. **3** The heat transfer coefficient also includes the radiation effects.

Properties The thermal conductivity is given to be k = 16 Btu/h.ft·°C.

Analysis The nodal spacing is given to be Δx=Δx=l=0.2 ft, and the general finite difference form of an interior node for steady two-dimensional heat conduction is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0$$



(a) There is symmetry about the vertical, horizontal, and diagonal lines passing through the center. Therefore, $T_1 = T_3 = T_7 = T_9$ and $T_2 = T_4 = T_6 = T_8$, and $T_1, T_2,$ and T_5 are the only 3 unknown nodal temperatures, and thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept for the interior nodes.

The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}: \quad 2k\frac{l}{2}\frac{T_2 - T_1}{l} + 2h\frac{l}{2}(T_\infty - T_1) + \frac{\dot{g}_0 l^2}{4} = 0$$

$$\text{Node 2 (convection)}: \quad 2k\frac{l}{2}\frac{T_1 - T_2}{l} + kl\frac{T_5 - T_2}{l} + hl(T_\infty - T_2) + \frac{\dot{g}_0 l^2}{2} = 0$$

$$\text{Node 5 (interior)}: \quad 4T_2 - 4T_5 + \frac{\dot{g}_0 l^2}{k} = 0$$

where $\dot{g}_0 = 0.19 \times 10^5 \text{ Btu/h}\cdot\text{ft}^3$, l = 0.2 ft, k = 16 Btu/h.ft·°F, h =7.9 Btu/h.ft^2·°F, and T_∞ =70°F. The 3 nodal temperatures under steady conditions are determined by solving the 3 equations above simultaneously with an equation solver to be

$T_1 = T_3 = T_7 = T_9$ **=304.85°F,**

$T_2 = T_4 = T_6 = T_8$ **=316.16°F,** T_5 =**328.04°F**

(b) The rate of heat loss from the bar through a 1-ft long section is determined from an energy balance on one-eight section of the bar, and multiplying the result by 8:

$$\dot{Q} = 8 \times \dot{Q}_{\text{one-eight section, conv}} = 8 \times \left[h\frac{l}{2}(T_1 - T_\infty) + h\frac{l}{2}(T_2 - T_\infty) \right](1\text{ ft}) = 8 \times h\frac{l}{2}\left[T_1 + T_2 - 2T_\infty\right](1\text{ ft})$$

$$= 8(7.9 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(0.2/2\text{ ft})(1\text{ft})[304.85 + 316.16 - 2\times 70]°\text{F}$$

$$= \mathbf{3040\ Btu/h} \quad \text{(per ft flength)}$$

Discussion Under steady conditions, the rate of heat loss from the bar is equal to the rate of heat generation within the bar per unit length, and is determined to be

$$\dot{Q} = \dot{E}_{\text{gen}} = \dot{g}_0 V = (0.19 \times 10^5 \text{ Btu/h.ft}^3)(0.4\text{ ft} \times 0.4\text{ ft} \times 1\text{ ft}) = 3040 \text{ Btu/h} \text{ (per ft length)}$$

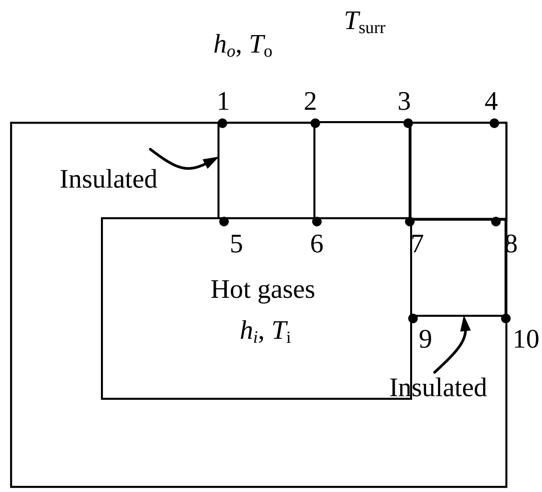
which confirms the results obtained by the finite difference method.

5-55 Heat transfer through a square chimney is considered. The nodal temperatures and the rate of heat loss per unit length are to be determined with the finite difference method.

Assumptions **1** Heat transfer is given to be steady and two-dimensional since the height of the chimney is large relative to its cross-section, and thus heat conduction through the chimney in the axial direction is negligible. It is tempting to simplify the problem further by considering heat transfer in each wall to be one dimensional which would be the case if the walls were thin and thus the corner effects were negligible. This assumption cannot be justified in this case since the walls are very thick and the corner sections constitute a considerable portion of the chimney structure. **2** There is no heat generation in the chimney. **3** Thermal conductivity is constant.

Properties The thermal conductivity and emissivity are given to be k = 1.4 W/m·°C and ε = 0.9.

Analysis (*a*) The most striking aspect of this problem is the apparent symmetry about the horizontal and vertical lines passing through the midpoint of the chimney. Therefore, we need to consider only one-fourth of the geometry in the solution whose nodal network consists of 10 equally spaced nodes. No heat can cross a symmetry line, and thus symmetry lines can be treated as insulated surfaces and thus "mirrors" in the finite-difference formulation. Considering a unit depth and using the energy balance approach for the boundary nodes (again assuming all heat transfer to be into the volume element for convenience), the finite difference formulation is obtained to be



Node 1: $h_0 \frac{l}{2}(T_0 - T_1) + k\frac{l}{2}\frac{T_2 - T_1}{l} + k\frac{l}{2}\frac{T_5 - T_1}{l} + \varepsilon\sigma\frac{l}{2}[T_{surr}^4 - (T_1 + 273)^4] = 0$

Node 2: $h_0 l(T_0 - T_2) + k\frac{l}{2}\frac{T_1 - T_2}{l} + k\frac{l}{2}\frac{T_3 - T_2}{l} + kl\frac{T_6 - T_2}{l} + \varepsilon\sigma l[T_{surr}^4 - (T_2 + 273)^4] = 0$

Node 3: $h_0 l(T_0 - T_3) + k\frac{l}{2}\frac{T_2 - T_3}{l} + k\frac{l}{2}\frac{T_4 - T_3}{l} + kl\frac{T_7 - T_3}{l} + \varepsilon\sigma l[T_{surr}^4 - (T_3 + 273)^4] = 0$

Node 4: $h_0 l(T_0 - T_4) + k\frac{l}{2}\frac{T_3 - T_4}{l} + k\frac{l}{2}\frac{T_8 - T_4}{l} + \varepsilon\sigma l[T_{surr}^4 - (T_4 + 273)^4] = 0$

Node 5: $h_i \frac{l}{2}(T_i - T_5) + k\frac{l}{2}\frac{T_6 - T_5}{l} + k\frac{l}{2}\frac{T_1 - T_5}{l} = 0$

Node 6: $h_i l(T_i - T_6) + k\frac{l}{2}\frac{T_5 - T_6}{l} + k\frac{l}{2}\frac{T_7 - T_6}{l} + kl\frac{T_2 - T_6}{l} = 0$

Node 7: $h_i l(T_i - T_7) + k\frac{l}{2}\frac{T_6 - T_7}{l} + k\frac{l}{2}\frac{T_9 - T_7}{l} + kl\frac{T_3 - T_7}{l} + kl\frac{T_8 - T_7}{l} = 0$

Node 8: $h_0 l(T_0 - T_8) + k\frac{l}{2}\frac{T_4 - T_8}{l} + k\frac{l}{2}\frac{T_{10} - T_8}{l} + kl\frac{T_7 - T_8}{l} + \varepsilon\sigma l[T_{surr}^4 - (T_8 + 273)^4] = 0$

Node 9: $h_i \frac{l}{2}(T_i - T_9) + k\frac{l}{2}\frac{T_7 - T_9}{l} + k\frac{l}{2}\frac{T_{10} - T_9}{l} = 0$

Node 10: $h_0 \frac{l}{2}(T_0 - T_{10}) + k\frac{l}{2}\frac{T_8 - T_{10}}{l} + k\frac{l}{2}\frac{T_9 - T_{10}}{l} + \varepsilon\sigma\frac{l}{2}[T_{surr}^4 - (T_{10} + 273)^4] = 0$

where $l = 0.1$ m, $k = 1.4$ W/m·°C, $h_i = 75$ W/m^2·°C, $T_i = 280$°C, $h_o = 18$ W/m^2·°C, $T_0 = 15$°C, $T_{surr} = 250$ K, $\varepsilon = 0.9$, and $\sigma = 5.67\times10^{-8}$ W/m^2.K^4. This system of 10 equations with 10 unknowns constitutes the finite difference formulation of the problem.

(*b*) The 10 nodal temperatures under steady conditions are determined by solving the 10 equations above simultaneously with an equation solver to be

$\boldsymbol{T_1 = 94.5°C}$, $\boldsymbol{T_2 = 93.0°C}$, $\boldsymbol{T_3 = 82.1°C}$, $\boldsymbol{T_4 = 36.1°C}$, $\boldsymbol{T_5 = 250.6°C}$,

$\boldsymbol{T_6 = 249.2°C}$, $\boldsymbol{T_7 = 229.7°C}$, $\boldsymbol{T_8 = 82.3°C}$, $\boldsymbol{T_9 = 261.5°C}$, $\boldsymbol{T_{10} = 94.6°C}$

(*c*) The rate of heat loss through a 1-m long section of the chimney is determined from

$$\begin{aligned}
\dot{Q} &= 4\sum \dot{Q}_{\text{one-fourth of chimney}} = 4\sum \dot{Q}_{\text{element, inner surface}} = 4\sum_m h_i A_{\text{surface},m}(T_i - T_m) \\
&= 4[h_i(l/2)(T_i - T_5) + h_i l(T_i - T_6) + h_i l(T_i - T_7) + h_i(l/2)(T_i - T_9)] \\
&= 4(75\ \text{W/m}^2\cdot°\text{C})(0.1\,\text{m}\times 1\,\text{m})[(280-250.6)/2 + (280-249.2) + (280-229.7) + (280-261.5)/2]°\text{C} \\
&= \mathbf{3153\ W}
\end{aligned}$$

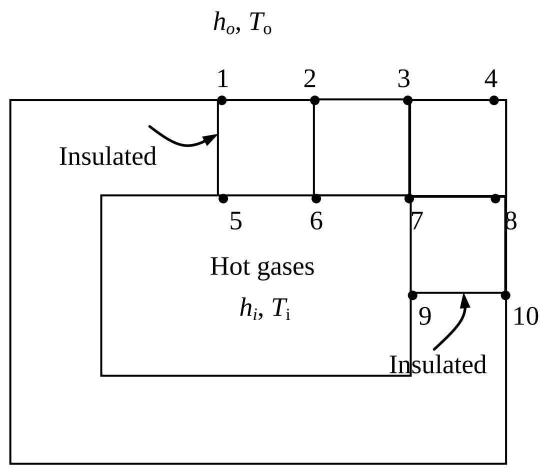
Discussion The rate of heat transfer can also be determined by calculating the heat loss from the outer surface by convection and radiation.

5-56 Heat transfer through a square chimney is considered. The nodal temperatures and the rate of heat loss per unit length are to be determined with the finite difference method.

Assumptions **1** Heat transfer is given to be steady and two-dimensional since the height of the chimney is large relative to its cross-section, and thus heat conduction through the chimney in the axial direction is negligible. It is tempting to simplify the problem further by considering heat transfer in each wall to be one dimensional which would be the case if the walls were thin and thus the corner effects were negligible. This assumption cannot be justified in this case since the walls are very thick and the corner sections constitute a considerable portion of the chimney structure. **2** There is no heat generation in the chimney. **3** Thermal conductivity is constant. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity of chimney is given to be k = 1.4 W/m·°C.

Analysis (*a*) The most striking aspect of this problem is the apparent symmetry about the horizontal and vertical lines passing through the midpoint of the chimney. Therefore, we need to consider only one-fourth of the geometry in the solution whose nodal network consists of 10 equally spaced nodes. No heat can cross a symmetry line, and thus symmetry lines can be treated as insulated surfaces and thus "mirrors" in the finite-difference formulation. Considering a unit depth and using the energy balance approach for the boundary nodes (again assuming all heat transfer to be into the volume element for convenience), the finite difference formulation is obtained to be



Node 1: $h_0 \frac{l}{2}(T_0 - T_1) + k\frac{l}{2}\frac{T_2 - T_1}{l} + k\frac{l}{2}\frac{T_5 - T_1}{l} = 0$

Node 2: $h_0 l(T_0 - T_2) + k\frac{l}{2}\frac{T_1 - T_2}{l} + k\frac{l}{2}\frac{T_3 - T_2}{l} + kl\frac{T_6 - T_2}{l} = 0$

Node 3: $h_0 l(T_0 - T_3) + k\frac{l}{2}\frac{T_2 - T_3}{l} + k\frac{l}{2}\frac{T_4 - T_3}{l} + kl\frac{T_7 - T_3}{l} = 0$

Node 4: $h_0 l(T_0 - T_4) + k\frac{l}{2}\frac{T_3 - T_4}{l} + k\frac{l}{2}\frac{T_8 - T_4}{l} = 0$

Node 5: $h_i \frac{l}{2}(T_i - T_5) + k\frac{l}{2}\frac{T_6 - T_5}{l} + k\frac{l}{2}\frac{T_1 - T_5}{l} = 0$

Node 6: $h_i l(T_i - T_6) + k\frac{l}{2}\frac{T_5 - T_6}{l} + k\frac{l}{2}\frac{T_7 - T_6}{l} + kl\frac{T_2 - T_6}{l} = 0$

Node 7: $h_i l(T_i - T_7) + k\frac{l}{2}\frac{T_6 - T_7}{l} + k\frac{l}{2}\frac{T_9 - T_7}{l} + kl\frac{T_3 - T_7}{l} + kl\frac{T_8 - T_7}{l} = 0$

Node 8: $h_0 l(T_0 - T_8) + k\frac{l}{2}\frac{T_4 - T_8}{l} + k\frac{l}{2}\frac{T_{10} - T_8}{l} + kl\frac{T_7 - T_8}{l} = 0$

Node 9: $h_i \frac{l}{2}(T_i - T_9) + k\frac{l}{2}\frac{T_7 - T_9}{l} + k\frac{l}{2}\frac{T_{10} - T_9}{l} = 0$

Node 10: $h_0 \frac{l}{2}(T_0 - T_{10}) + k\frac{l}{2}\frac{T_8 - T_{10}}{l} + k\frac{l}{2}\frac{T_9 - T_{10}}{l} = 0$

where l = 0.1 m, k = 1.4 W/m·°C, h_i = 75 W/m^2·°C, T_i =280°C, h_o = 18 W/m^2·°C, T_0 =15°C, and σ = 5.67×10^{-8} W/m^2.K^4. This system of 10 equations with 10 unknowns constitutes the finite difference formulation of the problem.

(*b*) The 10 nodal temperatures under steady conditions are determined by solving the 10 equations above simultaneously with an equation solver to be

$$\boldsymbol{T_1 = 118.8°C, \quad T_2 = 116.7°C, \quad T_3 = 103.4°C, \quad T_4 = 53.7°C, \quad T_5 = 254.4°C,}$$

$$\boldsymbol{T_6 = 253.0°C, \quad T_7 = 235.2°C, \quad T_8 = 103.5°C, \quad T_9 = 263.7°C, \quad T_{10} = 117.6°C}$$

(*c*) The rate of heat loss through a 1-m long section of the chimney is determined from

$$\begin{aligned}
\dot{Q} &= 4\sum \dot{Q}_{\text{one-fourth of chimney}} = 4\sum \dot{Q}_{\text{element, inner surface}} = 4\sum_m h_i A_{\text{surface},m}(T_i - T_m) \\
&= 4[h_i(l/2)(T_i - T_5) + h_i l(T_i - T_6) + h_i l(T_i - T_7) + h_i(l/2)(T_i - T_9)] \\
&= 4(75\ \text{W/m}^2\cdot°\text{C})(0.1\,\text{m}\times 1\,\text{m})[(280-254.4)/2 + (280-253.0) + (280-235.2) + (280-263.7)/2]°\text{C} \\
&= \mathbf{2783\ W}
\end{aligned}$$

Discussion The rate of heat transfer can also be determined by calculating the heat loss from the outer surface by convection.

5-57 "!PROBLEM 5-57"

"GIVEN"
k=1.4 "[W/m-C]"
A_flow=0.20*0.40 "[m^2]"
t=0.10 "[m]"
T_i=280 "[C], parameter to ve varied"
h_i=75 "[W/m^2-C]"
T_o=15 "[C]"
h_o=18 "[W/m^2-C]"
epsilon=0.9 "parameter to ve varied"
T_sky=250 "[K]"
DELTAx=0.10 "[m]"
DELTAy=0.10 "[m]"
d=1 "[m], unit depth is considered"
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"

"ANALYSIS"
"(b)"
l=DELTAx
"We consider only one-fourth of the geometry whose nodal network consists of 10 nodes. Using the finite difference method, 10 equations for 10 unknown temperatures are determined to be"
h_o*l/2*(T_o-T_1)+k*l/2*(T_2-T_1)/l+k*l/2*(T_5-T_1)/l+epsilon*sigma*l/2*(T_sky^4-(T_1+273)^4)=0 "Node 1"
h_o*l*(T_o-T_2)+k*l/2*(T_1-T_2)/l+k*l/2*(T_3-T_2)/l+k*l*(T_6-T_2)/l+epsilon*sigma*l*(T_sky^4-(T_2+273)^4)=0 "Node 2"
h_o*l*(T_o-T_3)+k*l/2*(T_2-T_3)/l+k*l/2*(T_4-T_3)/l+k*l*(T_7-T_3)/l+epsilon*sigma*l*(T_sky^4-(T_3+273)^4)=0 "Node 3"
h_o*l*(T_o-T_4)+k*l/2*(T_3-T_4)/l+k*l/2*(T_8-T_4)/l+epsilon*sigma*l*(T_sky^4-(T_4+273)^4)=0 "Node 4"
h_i*l/2*(T_i-T_5)+k*l/2*(T_6-T_5)/l+k*l/2*(T_1-T_5)/l=0 "Node 5"
h_i*l*(T_i-T_6)+k*l/2*(T_5-T_6)/l+k*l/2*(T_7-T_6)/l+k*l*(T_2-T_6)/l=0 "Node 6"
h_i*l*(T_i-T_7)+k*l/2*(T_6-T_7)/l+k*l/2*(T_9-T_7)/l+k*l*(T_3-T_7)/l+k*l*(T_8-T_7)/l=0 "Node 7"
h_o*l*(T_o-T_8)+k*l/2*(T_4-T_8)/l+k*l/2*(T_10-T_8)/l+k*l*(T_7-T_8)/l+epsilon*sigma*l*(T_sky^4-(T_8+273)^4)=0 "Node 8"
h_i*l*(T_i-T_9)+k*l/2*(T_7-T_9)/l+k*l/2*(T_10-T_9)/l=0 "Node 9"
h_o*l/2*(T_o-T_10)+k*l/2*(T_8-T_10)/l+k*l/2*(T_9-T_10)/l+epsilon*sigma*l/2*(T_sky^4-(T_10+273)^4)=0 "Node 10"
"Right top corner is considered. The locations of nodes are as follows:"
"Node 1: Middle of top surface
Node 2: At the right side of node 1
Node 3: At the right side of node 2
Node 4: Corner node
Node 5: The node below node 1, at the middle of inner top surface
Node 6: The node below node 2
Node 7: The node below node 3, at the inner corner
Node 8: The node below node 4
Node 9: The node below node 7,at the middle of inner right surface
Node 10: The node below node 8, at the middle of outer right surface"
T_corner=T_4
T_inner_middle=T_9
"(c)"
"The rate of heat loss through a unit depth d=1 m of the chimney is"
Q_dot=4*(h_i*l/2*d*(T_i-T_5)+h_i*l*d*(T_i-T_6)+h_i*l*d*(T_i-T_7)+h_i*l/2*d*(T_i-T_9))

T_i [C]	T_{corner} [C]	$T_{inner, middle}$ [C]	Q [W]
200	28.38	187	2206
210	29.37	196.3	2323
220	30.35	205.7	2441

230	31.32	215	2559
240	32.28	224.3	2677
250	33.24	233.6	2796
260	34.2	242.9	2914
270	35.14	252.2	3033
280	36.08	261.5	3153
290	37.02	270.8	3272
300	37.95	280.1	3392
310	38.87	289.3	3512
320	39.79	298.6	3632
330	40.7	307.9	3752
340	41.6	317.2	3873
350	42.5	326.5	3994
360	43.39	335.8	4115
370	44.28	345.1	4237
380	45.16	354.4	4358
390	46.04	363.6	4480
400	46.91	372.9	4602

ε	T_{corner} **[C]**	$T_{inner, middle}$ **[C]**	**Q [W]**
0.1	51.09	263.4	2836
0.15	49.87	263.2	2862
0.2	48.7	263.1	2886
0.25	47.58	262.9	2909
0.3	46.5	262.8	2932
0.35	45.46	262.7	2953
0.4	44.46	262.5	2974
0.45	43.5	262.4	2995
0.5	42.56	262.3	3014
0.55	41.66	262.2	3033
0.6	40.79	262.1	3052
0.65	39.94	262	3070
0.7	39.12	261.9	3087
0.75	38.33	261.8	3104
0.8	37.56	261.7	3121
0.85	36.81	261.6	3137
0.9	36.08	261.5	3153
0.95	35.38	261.4	3168
1	34.69	261.3	3183

5-58 The exposed surface of a long concrete damn of triangular cross-section is subjected to solar heat flux and convection and radiation heat transfer. The vertical section of the damn is subjected to convection with water. The temperatures at the top, middle, and bottom of the exposed surface of the damn are to be determined.

Assumptions **1** Heat transfer through the damn is given to be steady and two-dimensional. **2** There is no heat generation within the damn. **3** Heat transfer through the base is negligible. **4** Thermal properties and heat transfer coefficients are constant.

Properties The thermal conductivity and solar absorptivity are given to be k = 0.6 W/m·°C and α_s = 0.7.

Analysis The nodal spacing is given to be Δx=Δx=l=1 m, and all nodes are boundary nodes. Node 5 on the insulated boundary can be treated as an interior node for which $T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0$. Using the energy balance approach and taking the direction of all heat transfer to be towards the node, the finite difference equations for the nodes are obtained to be as follows:

Node 1: $$h_i \frac{l}{2}(T_i - T_1) + k\frac{l}{2}\frac{T_2 - T_1}{l} + \frac{l/2}{\sin 45}\left[\alpha_s \dot{q}_s + h_0(T_0 - T_1)\right] = 0$$

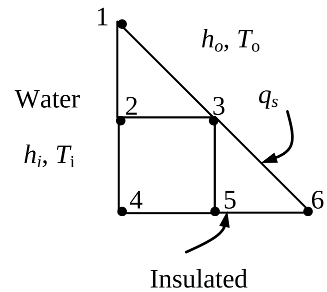
Node 2: $$h_i l(T_i - T_1) + k\frac{l}{2}\frac{T_1 - T_2}{l} + k\frac{l}{2}\frac{T_4 - T_2}{l} + kl\frac{T_3 - T_2}{l} = 0$$

Node 3: $$kl\frac{T_2 - T_3}{l} + kl\frac{T_5 - T_3}{l} + \frac{l}{\sin 45}\left[\alpha_s \dot{q}_s + h_0(T_0 - T_3)\right] = 0$$

Node 4: $$h_i \frac{l}{2}(T_i - T_4) + k\frac{l}{2}\frac{T_2 - T_4}{l} + k\frac{l}{2}\frac{T_5 - T_4}{l} = 0$$

Node 5: $$T_4 + 2T_3 + T_6 - 4T_5 = 0$$

Node 6: $$k\frac{l}{2}\frac{T_5 - T_6}{l} + \frac{l/2}{\sin 45}\left[\alpha_s \dot{q}_s + h_0(T_0 - T_6)\right] = 0$$



where l = 1 m, k = 0.6 W/m·°C, h_i =150 W/m^2·°C, T_i =15°C, h_o = 30 W/m^2·°C, T_0 =25°C, α_s = 0.7, and $\dot{q}_s = 800\ \text{W/m}^2$. The system of 6 equations with 6 unknowns constitutes the finite difference formulation of the problem. The 6 nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

$T_1 = T_{top}$ =**21.3**°C, T_2 =15.1°C, $T_3 = T_{middle}$ =**43.2**°C, T_4 =15.1°C, T_5 =36.3°C, $T_6 = T_{bottom}$ =**43.6**°C

Discussion Note that the highest temperature occurs at a location furthest away from the water, as expected.

5-59E The top and bottom surfaces of a V-grooved long solid bar are maintained at specified temperatures while the left and right surfaces are insulated. The temperature at the middle of the insulated surface is to be determined.

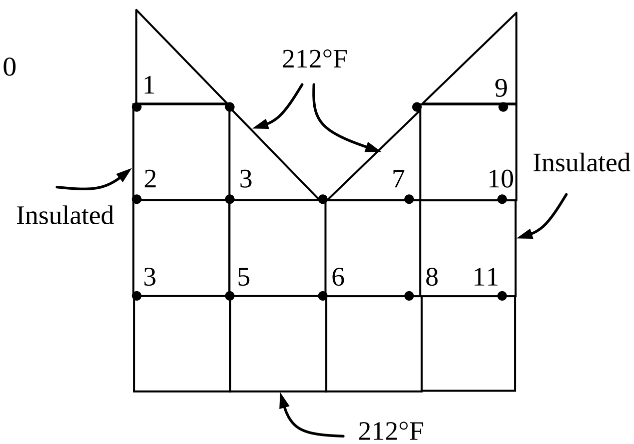
Assumptions 1 Heat transfer through the bar is given to be steady and two-dimensional. **2** There is no heat generation within the bar. **3** Thermal properties are constant.

Analysis The nodal spacing is given to be $\Delta x = \Delta y = l = 1$ ft, and the general finite difference form of an interior node for steady two-dimensional heat conduction with no heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0 \;\rightarrow\; T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0$$

There is symmetry about the vertical plane passing through the center. Therefore, $T_1 = T_9$, $T_2 = T_{10}$, $T_3 = T_{11}$, $T_4 = T_7$, and $T_5 = T_8$. Therefore, there are only 6 unknown nodal temperatures, and thus we need only 6 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.

The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:



Node 1: $k\frac{l}{2}\frac{32-T_1}{l} + kl\frac{32-T_1}{l} + k\frac{l}{2}\frac{T_2-T_1}{l} = 0$

(Note that k and l cancel out)

Node 2: $T_1 + 2T_4 + T_3 - 4T_2 = 0$

Node 3: $T_2 + 212 + 2T_5 - 4T_3 = 0$

Node 4: $2\times 32 + T_2 + T_5 - 4T_4 = 0$

Node 5: $T_3 + 212 + T_4 + T_6 - 4T_5 = 0$

Node 6: $32 + 212 + 2T_5 - 4T_6 = 0$

The 6 nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

$T_1 = 44.7$°F, $T_2 = 82.8$°F, $T_3 = 143.4$°F, $T_4 = 71.6$°F, $T_5 = 139.4$°F, $T_6 = 130.7$°F

Therefore, the temperature at the middle of the insulated surface will be $\boldsymbol{T_2 = 82.8}$**°F.**

5-60 "!PROBLEM 5-60E"

"GIVEN"
T_top=32 "[F], parameter to be varied"
T_bottom=212 "[F], parameter to be varied"
DELTAx=1 "[ft]"
DELTAy=1 "[ft]"

"ANALYSIS"
l=DELTAx
T_1=T_9 "due to symmetry"
T_2=T_10 "due to symmetry"
T_3=T_11 "due to symmetry"
T_4=T_7 "due to symmetry"
T_5=T_8 "due to symmetry"
"Using the finite difference method, the six equations for the six unknown temperatures are determined to be"
"k*l/2*(T_top-T_1)/l+k*l*(T_top-T_1)/l+k*l/2*(T_2-T_1)/l=0 simplifies to for Node 1"
1/2*(T_top-T_1)+(T_top-T_1)+1/2*(T_2-T_1)=0 "Node 1"
T_1+2*T_4+T_3-4*T_2=0 "Node 2"
T_2+T_bottom+2*T_5-4*T_3=0 "Node 3"
2*T_top+T_2+T_5-4*T_4=0 "Node 4"
T_3+T_bottom+T_4+T_6-4*T_5=0 "Node 5"
T_top+T_bottom+2*T_5-4*T_6=0 "Node 6"

T_{top} [F]	T_2 [F]
32	82.81
41.47	89.61
50.95	96.41
60.42	103.2
69.89	110
79.37	116.8
88.84	123.6
98.32	130.4
107.8	137.2
117.3	144
126.7	150.8
136.2	157.6
145.7	164.4
155.2	171.2
164.6	178
174.1	184.8
183.6	191.6
193.1	198.4
202.5	205.2
212	212

T_{bottom} [F]	T_2 [F]
32	32
41.47	34.67
50.95	37.35
60.42	40.02
69.89	42.7
79.37	45.37
88.84	48.04
98.32	50.72
107.8	53.39
117.3	56.07
126.7	58.74
136.2	61.41
145.7	64.09
155.2	66.76
164.6	69.44
174.1	72.11
183.6	74.78
193.1	77.46
202.5	80.13
212	82.81

5-61 The top and bottom surfaces of an L-shaped long solid bar are maintained at specified temperatures while the left surface is insulated and the remaining 3 surfaces are subjected to convection. The finite difference formulation of the problem is to be obtained, and the unknown nodal temperatures are to be determined.

Assumptions **1** Heat transfer through the bar is given to be steady and two-dimensional. **2** There is no heat generation within the bar. **3** Thermal properties and heat transfer coefficients are constant. **4** Radiation heat transfer is negligible.

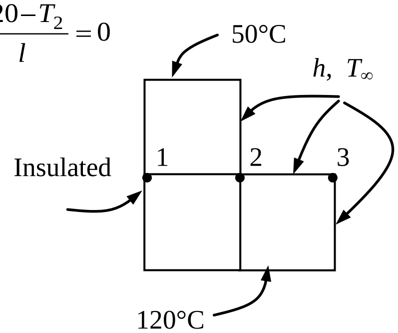
Properties The thermal conductivity is given to be k = 12 W/m·°C.

Analysis (a) The nodal spacing is given to be Δx=Δx=l=0.1 m, and all nodes are boundary nodes. Node 1 on the insulated boundary can be treated as an interior node for which $T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0$. Using the energy balance approach and taking the direction of all heat transfer to be towards the node, the finite difference equations for the nodes are obtained to be as follows:

Node 1: $50 + 120 + 2T_2 - 4T_1 = 0$

Node 2: $hl(T_\infty - T_2) + k\dfrac{l}{2}\dfrac{50 - T_2}{l} + k\dfrac{l}{2}\dfrac{T_3 - T_2}{l} + kl\dfrac{T_1 - T_2}{l} + kl\dfrac{120 - T_2}{l} = 0$

Node 3: $hl(T_\infty - T_3) + k\dfrac{l}{2}\dfrac{T_2 - T_3}{l} + k\dfrac{l}{2}\dfrac{120 - T_3}{l} = 0$



where l = 0.1 m, k = 12 W/m·°C, h =30 W/m^2·°C, and T_∞ =25°C. This system of 3 equations with 3 unknowns constitute the finite difference formulation of the problem.

(b) The 3 nodal temperatures under steady conditions are determined by solving the 3 equations above simultaneously with an equation solver to be

$\boldsymbol{T_1}$ **= 85.7°C,** $\boldsymbol{T_2}$ **=86.4°C,** $\boldsymbol{T_3}$ **=87.6°C**

5-62 A rectangular block is subjected to uniform heat flux at the top, and iced water at 0°C at the sides. The steady finite difference formulation of the problem is to be obtained, and the unknown nodal temperatures as well as the rate of heat transfer to the iced water are to be determined.

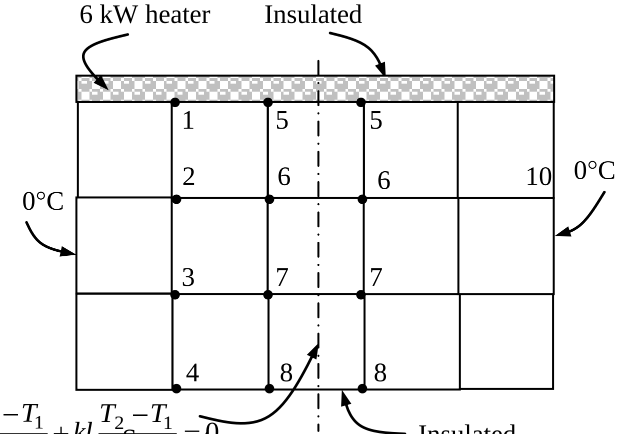
Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** There is no heat generation within the block. **3** The heat transfer coefficient is very high so that the temperatures on both sides of the block can be taken to be 0°C. **4** Heat transfer through the bottom surface is negligible.

Properties The thermal conductivity is given to be k = 23 W/m·°C.

Analysis The nodal spacing is given to be $\Delta x=\Delta x=l=0.1$ m, and the general finite difference form of an interior node equation for steady 2-D heat conduction is expressed as

$$T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}+\frac{\dot{g}_{\text{node}}l^2}{k}=0$$

$$T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}=0$$



There is symmetry about a vertical line passing through the middle of the region, and we need to consider only half of the region. Note that all side surfaces are at T_0 = 0°C, and there are 8 nodes with unknown temperatures. Replacing the symmetry lines by insulation and utilizing the mirror-image concept, the finite difference equations are obtained to be as follows:

Node 1 (heat flux): $\dot{q}_0 l + k\frac{l}{2}\frac{T_0-T_1}{l}+k\frac{l}{2}\frac{T_5-T_1}{l}+kl\frac{T_2-T_1}{l}=0$

Node 2 (interior): $T_0+T_1+T_3+T_6-4T_2=0$

Node 3 (interior): $T_0+T_2+T_4+T_7-4T_3=0$

Node 4 (insulation): $T_0+2T_3+T_8-4T_4=0$

Node 5 (heat flux): $\dot{q}_0 l + k\frac{l}{2}\frac{T_1-T_5}{l}+kl\frac{T_6-T_5}{l}+0=0$

Node 6 (interior): $T_2+T_5+T_6+T_7-4T_6=0$

Node 7 (interior): $T_3+T_6+T_7+T_8-4T_7=0$

Node 8 (insulation): $T_4+2T_7+T_8-4T_8=0$

where l = 0.1 m, k = 23 W/m·°C, T_0 =0°C, and $\dot{q}_0=\dot{Q}_0/A=(6000\,\text{W})/(5\times 0.5\,\text{m}^2)=2400\,\text{W/m}^2$. This system of 8 equations with 8 unknowns constitutes the finite difference formulation of the problem.

(b) The 8 nodal temperatures under steady conditions are determined by solving the 8 equations above simultaneously with an equation solver to be

T_1 =13.7°C, T_2 =7.4°C, T_3 =4.7°C, T_4 =3.9°C, T_5 =19.0°C, T_6 =11.3°C, T_7 =7.4°C, T_8 =6.2°C

(c) The rate of heat transfer from the block to the iced water is 6 kW since all the heat supplied to the block from the top must be equal to the heat transferred from the block. Therefore, $\dot{Q}=\mathbf{6\,kW}$.

Discussion The rate of heat transfer can also be determined by calculating the heat loss from the side surfaces using the heat conduction relation.