5-84 A uranium plate initially at a uniform temperature is subjected to insulation on one side and convection on the other. The transient finite difference formulation of this problem is to be obtained, and the nodal temperatures after 5 min and under steady conditions are to be determined.

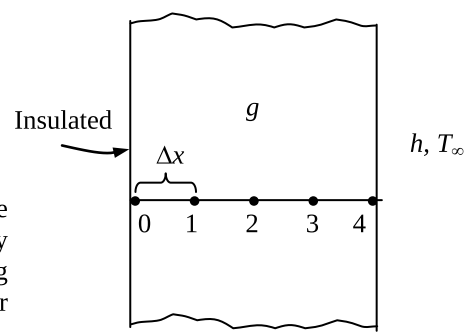
Assumptions **1** Heat transfer is one-dimensional since the plate is large relative to its thickness. **2** Thermal conductivity is constant. **3** Radiation heat transfer is negligible.

Properties The conductivity and diffusivity are given to be k = 28 W/m·°C and $\alpha = 12.5\times10^{-6}\ \text{m}^2/\text{s}$.

Analysis The nodal spacing is given to be Δx = 0.02 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.08/0.02+1 = 5. This problem involves 5 unknown nodal temperatures, and thus we need to have 5 equations. Node 0 is on insulated boundary, and thus we can treat it as an interior note by using the mirror image concept. Nodes 1, 2, and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{g}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$\rightarrow T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{g}_m^i \Delta x^2}{k}$$



The finite difference equation for node 4 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about node 4 and taking the direction of all heat transfers to be towards the node under consideration:

Node 0 (insulated): $$T_0^{i+1} = \tau(T_1^i + T_1^i) + (1-2\tau)T_0^i + \tau\frac{\dot{g}_0 \Delta x^2}{k}$$

Node 1 (interior): $$T_1^{i+1} = \tau(T_0^i + T_2^i) + (1-2\tau)T_1^i + \tau\frac{\dot{g}_0 \Delta x^2}{k}$$

Node 2 (interior): $$T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i + \tau\frac{\dot{g}_0 \Delta x^2}{k}$$

Node 3 (interior): $$T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i + \tau\frac{\dot{g}_0 \Delta x^2}{k}$$

Node 4 (convection): $$h(T_\infty - T_4^i) + k\frac{T_3^i - T_4^i}{\Delta x} + \dot{g}_0\frac{\Delta x}{2} = \rho\frac{\Delta x}{2}C\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

or $$T_4^{i+1} = \left(1 - 2\tau - 2\tau\frac{h\Delta x}{k}\right)T_4^i + 2\tau T_3^i + 2\tau\frac{h\Delta x}{k}T_\infty + \tau\frac{\dot{g}_0(\Delta x)^2}{k}$$

where $\Delta x = 0.02\,\text{m}, \dot{g}_0 = 10^6\ \text{W/m}^3,\ k = 28\,\text{W/m}\cdot°\text{C},\ h = 35\,\text{W/m}^2\cdot°\text{C},\ T_\infty = 20°\text{C}$, and $\alpha = 12.5\times10^{-6}$ m^2/s. The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient of T_4^i is smaller in this case, and thus the stability criteria for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h\Delta x}{k} \geq 0 \quad \rightarrow \quad \tau \leq \frac{1}{2(1 + h\Delta x/k)} \quad \rightarrow \quad \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h\Delta x/k)}$$

since $\tau = \alpha\Delta t/\Delta x^2$. Substituting the given quantities, the maximum allowable the time step becomes

$$\Delta t \leq \frac{(0.02\,\text{m})^2}{2(12.5\times10^{-6}\ \text{m}^2/\text{s})[1 + (35\,\text{W/m}^2.°\text{C})(0.02\,\text{m})/(28\,\text{W/m}.°\text{C})]} = 15.6\,\text{s}$$

Therefore, any time step less than 15.5 s can be used to solve this problem. For convenience, let us choose the time step to be Δt = 15 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha \Delta t}{\Delta x^2} = \frac{(12.5 \times 10^{-6}\ \text{m}^2/\text{s})(15\ \text{s})}{(0.02\ \text{m})^2} = 0.46875$$

Substituting this value of τ and other given quantities, the nodal temperatures after 5×60/15 = 20 time steps (5 min) are determined to be

After 5 min: $\boldsymbol{T_0}$ **=228.9°C,** $\boldsymbol{T_1}$ **=228.4°C,** $\boldsymbol{T_2}$ **=226.8°C,** $\boldsymbol{T_3}$ **=224.0°C, and** $\boldsymbol{T_4}$ **=219.9 °C**

(*b*) The time needed for transient operation to be established is determined by increasing the number of time steps until the nodal temperatures no longer change. In this case steady operation is established in ---- min, and the nodal temperatures under steady conditions are determined to be

$\boldsymbol{T_0}$ **=2420°C,** $\boldsymbol{T_1}$ **=2413°C,** $\boldsymbol{T_2}$ **=2391°C,** $\boldsymbol{T_3}$ **=2356°C, and** $\boldsymbol{T_4}$ **=2306 °C**

Discussion The steady solution can be checked independently by obtaining the steady finite difference formulation, and solving the resulting equations simultaneously.

5-85 "!PROBLEM 5-85"

"GIVEN"
L=0.08 "[m]"
k=28 "[W/m-C]"
alpha=12.5E-6 "[m^2/s]"
T_i=100 "[C]"
g_dot=1E6 "[W/m^3]"
T_infinity=20 "[C]"
h=35 "[W/m^2-C]"
DELTAx=0.02 "[m]"
"time=300 [s], parameter to be varied"

"ANALYSIS"
M=L/DELTAx+1 "Number of nodes"
DELTAt=15 "[s]"
tau=(alpha*DELTAt)/DELTAx^2

"The technique is to store the temperatures in the parametric table and recover them (as old temperatures)
using the variable ROW. The first row contains the initial values so Solve Table must begin at row 2.
Use the DUPLICATE statement to reduce the number of equations that need to be typed. Column 1
contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 7 the Row."
Time=TableValue(Row-1,#Time)+DELTAt
Duplicate i=1,5
    T_old[i]=TableValue(Row-1,#T[i])
end

"Using the explicit finite difference approach, the six equations for the six unknown temperatures are determined to be"
T[1]=tau*(T_old[2]+T_old[2])+(1-2*tau)*T_old[1]+tau*(g_dot*DELTAx^2)/k "Node 1, insulated"
T[2]=tau*(T_old[1]+T_old[3])+(1-2*tau)*T_old[2]+tau*(g_dot*DELTAx^2)/k "Node 2"
T[3]=tau*(T_old[2]+T_old[4])+(1-2*tau)*T_old[3]+tau*(g_dot*DELTAx^2)/k "Node 3"
T[4]=tau*(T_old[3]+T_old[5])+(1-2*tau)*T_old[4]+tau*(g_dot*DELTAx^2)/k "Node 4"
T[5]=(1-2*tau-2*tau*(h*DELTAx)/k)*T_old[5]+2*tau*T_old[4]+2*tau*(h*DELTAx)/k*T_infinity+tau*(g_dot*DELTAx^2)/k "Node 4, convection"

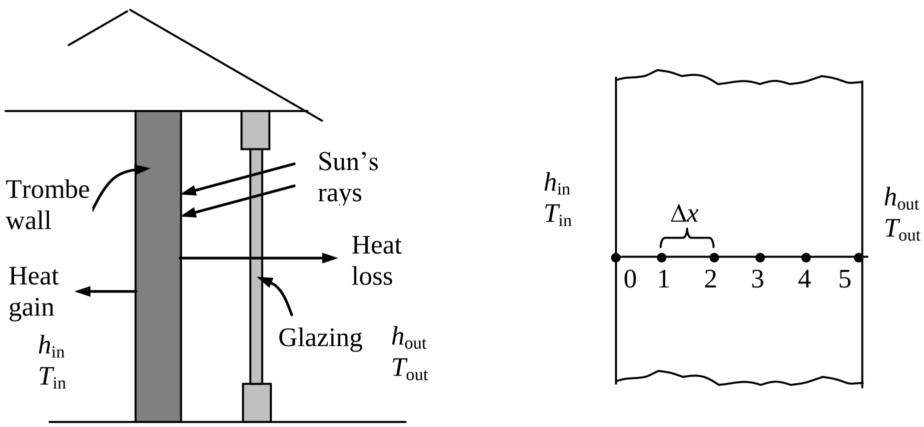
Time [s]	**T_1 [C]**	**T_2 [C]**	**T_3 [C]**	**T_4 [C]**	**T_5 [C]**	**Row**
0	100	100	100	100	100	1
15	106.7	106.7	106.7	106.7	104.8	2
30	113.4	113.4	113.4	112.5	111.3	3
45	120.1	120.1	119.7	119	117	4
60	126.8	126.6	126.3	125.1	123.3	5
75	133.3	133.2	132.6	131.5	129.2	6
90	139.9	139.6	139.1	137.6	135.5	7
105	146.4	146.2	145.4	144	141.5	8
120	152.9	152.6	151.8	150.2	147.7	9
135	159.3	159.1	158.1	156.5	153.7	10
…	…	…	…	…	…	…
…	…	…	…	…	…	…
3465	1217	1213	1203	1185	1160	232
3480	1220	1216	1206	1188	1163	233
3495	1223	1220	1209	1192	1167	234
3510	1227	1223	1213	1195	1170	235
3525	1230	1227	1216	1198	1173	236
3540	1234	1230	1219	1201	1176	237
3555	1237	1233	1223	1205	1179	238
3570	1240	1237	1226	1208	1183	239
3585	1244	1240	1229	1211	1186	240
3600	1247	1243	1233	1214	1189	241

5-86 The passive solar heating of a house through a Trombe wall is studied. The temperature distribution in the wall in 12 h intervals and the amount of heat transfer during the first and second days are to be determined.

Assumptions **1** Heat transfer is one-dimensional since the exposed surface of the wall large relative to its thickness. **2** Thermal conductivity is constant. **3** The heat transfer coefficients are constant.

Properties The wall properties are given to be k = 0.70 W/m·°C, $\alpha = 0.44\times10^{-6}$ m^2/s, and $\kappa = 0.76$. The hourly variation of monthly average ambient temperature and solar heat flux incident on a vertical surface is given to be

Time of day	Ambient Temperature, °C	Solar insolation W/m^2
7am-10am	0	375
10am-1pm	4	750
1pm-4pm	6	580
4pm-7pm	1	95
7pm-10pm	-2	0
10pm-1am	-3	0
1am-4am	-4	0
4am-7am	-4	0



Analysis The nodal spacing is given to be Δx = 0.05 m, Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.30/0.05+1 = 7. This problem involves 7 unknown nodal temperatures, and thus we need to have 7 equations. Nodes 1, 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{g}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau} \;\rightarrow\; T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i$$

The finite difference equation for boundary nodes 0 and 6 are obtained by applying an energy balance on the half volume elements and taking the direction of all heat transfers to be towards the node under consideration:

Node 0: $h_{in} A(T_{in}^i - T_0^i) + kA\dfrac{T_1^i - T_0^i}{\Delta x} = \rho A \dfrac{\Delta x}{2} C \dfrac{T_0^{i+1} - T_0^i}{\Delta t}$

or $\quad T_0^{i+1} = \left(1 - 2\tau - 2\tau \dfrac{h_{in}\Delta x}{k}\right) T_0^i + 2\tau T_1^i + 2\tau \dfrac{h_{in}\Delta x}{k} T_{in}$

Node 1 $(m=1)$: $T_1^{i+1} = \tau(T_0^i + T_2^i) + (1-2\tau)T_1^i$

Node 2 $(m=2)$: $T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i$

Node 3 $(m=3)$: $T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i$

Node 4 $(m=4)$: $T_4^{i+1} = \tau(T_3^i + T_5^i) + (1-2\tau)T_4^i$

Node 5 $(m=5)$: $T_5^{i+1} = \tau(T_4^i + T_6^i) + (1-2\tau)T_5^i$

Node 6 $$h_{\text{out}} A(T_{\text{out}}^i - T_6^i) + \kappa A\dot{q}_{\text{solar}}^i + kA\frac{T_5^i - T_6^i}{\Delta x} = \rho A\frac{\Delta x}{2}C\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

or $$T_6^{i+1} = \left(1 - 2\tau - 2\tau\frac{h_{\text{out}}\Delta x}{k}\right)T_6^i + 2\tau T_5^i + 2\tau\frac{h_{\text{out}}\Delta x}{k}T_{\text{out}}^i + 2\tau\frac{\kappa\dot{q}_{\text{solar}}^i\Delta x}{k}$$

where L = 0.30 m, k = 0.70 W/m.°C, $\alpha = 0.44\times10^{-6}\ \text{m}^2/\text{s}$, T_{out} and $\dot{q}_{\text{solar}}$ are as given in the table, $\kappa = 0.76$ h_{out} = 3.4 W/m^2.°C, T_{in} = 20°C, h_{in} = 9.1 W/m^2.°C, and Δx = 0.05 m.

Next we need to determine the upper limit of the time step Δt from the stability criteria since we are using the explicit method. This requires the identification of the smallest primary coefficient in the system. We know that the boundary nodes are more restrictive than the interior nodes, and thus we examine the formulations of the boundary nodes 0 and 6 only. The smallest and thus the most restrictive primary coefficient in this case is the coefficient of T_0^i in the formulation of node 0 since $h_{\text{in}} > h_{\text{out}}$, and thus

$$1 - 2\tau - 2\tau\frac{h_{\text{in}}\Delta x}{k} < 1 - 2\tau - 2\tau\frac{h_{\text{out}}\Delta x}{k}$$

Therefore, the stability criteria for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h_{\text{in}}\Delta x}{k} \geq 0 \quad \rightarrow \quad \tau \leq \frac{1}{2(1 + h_{\text{in}}\Delta x/k)} \quad \rightarrow \quad \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h_{\text{in}}\Delta x/k)}$$

since $\tau = \alpha\Delta t/\Delta x^2$. Substituting the given quantities, the maximum allowable the time step becomes

$$\Delta t \leq \frac{(0.05\,\text{m})^2}{2(0.44\times10^{-6}\ \text{m}^2/\text{s})[1 + (9.1\ \text{W/m}^2.°\text{C})(0.05\,\text{m})/(0.70\ \text{W/m.°C})]} = 1722\,\text{s}$$

Therefore, any time step less than 1722 s can be used to solve this problem. For convenience, let us choose the time step to be Δt = 900 s = 15 min. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(0.44\times10^{-6}\ \text{m}^2/\text{s})(900\,\text{s})}{(0.05\,\text{m})^2} = 0.1584$$

Initially (at 7 am or t = 0), the temperature of the wall is said to vary linearly between 20°C at node 0 and 0°C at node 6. Noting that there are 6 nodal spacing of equal length, the temperature change between two neighboring nodes is (20 - 0)°F/6 = 3.33°C. Therefore, the initial nodal temperatures are

$$T_0^0 = 20°\text{C},\ T_1^0 = 16.66°\text{C},\ T_2^0 = 13.33°\text{C},\ T_3^0 = 10°\text{C},\ T_4^0 = 6.66°\text{C},\ T_5^0 = 3.33°\text{C},\ T_6^0 = 0°\text{C}$$

Substituting the given and calculated quantities, the nodal temperatures after 6, 12, 18, 24, 30, 36, 42, and 48 h are calculated and presented in the following table and chart.

Time	Time step, i	Nodal temperatures, °C						
		T_0	T_1	T_2	T_3	T_4	T_5	T_6
0 h (7am)	0	20.0	16.7	13.3	10.0	6.66	3.33	0.0
6 h (1 pm)	24	17.5	16.1	15.9	18.1	24.8	38.8	61.5
12 h (7 pm)	48	21.4	22.9	25.8	30.2	34.6	37.2	35.8
18 h (1 am)	72	22.9	24.6	26.0	26.6	26.0	23.5	19.1
24 h (7 am)	96	21.6	22.5	22.7	22.1	20.4	17.7	13.9
30 h (1 pm)	120	21.0	21.8	23.4	26.8	34.1	47.6	68.9
36 h (7 pm)	144	24.1	27.0	31.3	36.4	41.1	43.2	40.9
42 h (1 am)	168	24.7	27.6	29.9	31.1	30.5	27.8	22.6
48 h (7 am)	192	23.0	24.6	25.5	25.2	23.7	20.7	16.3

The rate of heat transfer from the Trombe wall to the interior of the house during each time step is determined from Newton's law of cooling using the average temperature at the inner surface of the wall (node 0) as

$$Q^i_{\text{Trumbe wall}} = \dot{Q}^i_{\text{Trumbe wall}}\Delta t = h_{\text{in}}A(T^i_0 - T_{\text{in}})\Delta t = h_{\text{in}}A[(T^i_0 + T^{i-1}_0)/2 - T_{\text{in}}]\Delta t$$

Therefore, the amount of heat transfer during the first time step (i = 1) or during the first 15 min period is

$$Q^1_{\text{Trumbewall}} = h_{\text{in}}A[(T^1_0 + T^0_0)/2 - T_{\text{in}}]\Delta t = (9.1\,\text{W/m}^2.°\text{C})(2.8\times 7\,\text{m}^2)[(68.3+70)/2 - 70°\text{F}](0.25\,\text{h}) = -96.8\,\text{Btu}$$

The negative sign indicates that heat is transferred to the Trombe wall from the air in the house which represents a heat loss. Then the total heat transfer during a specified time period is determined by adding the heat transfer amounts for each time step as

$$Q_{\text{Trumbe wall}} = \sum_{i=1}^{I} Q^i_{\text{Trumbe wall}} = \sum_{i=1}^{I} h_{\text{in}}A[(T^i_0 + T^{i-1}_0)/2 - T_{\text{in}}]\Delta t$$

where I is the total number of time intervals in the specified time period. In this case I = 48 for 12 h, 96 for 24 h, etc. Following the approach described above using a computer, the amount of heat transfer between the Trombe wall and the interior of the house is determined to be

$Q_{\text{Trombe wall}}$ = - 3421 kJ after 12 h

$Q_{\text{Trombe wall}}$ = 1753 kJ after 24 h

$Q_{\text{Trombe wall}}$ = 5393 kJ after 36 h

$Q_{\text{Trombe wall}}$ = 15,230 kJ after 48 h

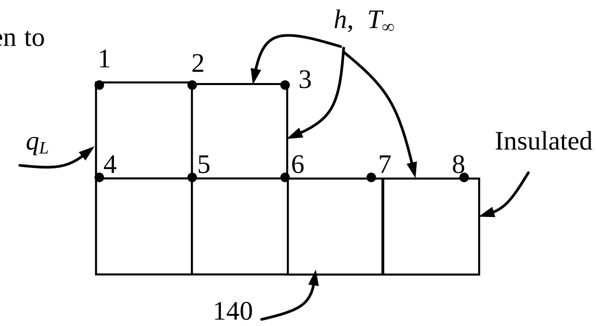
Discussion Note that the interior temperature of the Trombe wall drops in early morning hours, but then rises as the solar energy absorbed by the exterior surface diffuses through the wall. The exterior surface temperature of the Trombe wall rises from 0 to 61.5°C in just 6 h because of the solar energy absorbed, but then drops to 13.9°C by next morning as a result of heat loss at night. Therefore, it may be worthwhile to cover the outer surface at night to minimize the heat losses.

Also the house loses 3421 kJ through the Trombe wall the 1st daytime as a result of the low start-up temperature, but delivers about 13,500 kJ of heat to the house the second day. It can be shown that the Trombe wall will deliver even more heat to the house during the 3rd day since it will start the day at a higher average temperature.

5-87 Heat conduction through a long L-shaped solid bar with specified boundary conditions is considered. The temperature at the top corner (node #3) of the body after 2, 5, and 30 min is to be determined with the transient explicit finite difference method.

Assumptions **1** Heat transfer through the body is given to be transient and two-dimensional. **2** Thermal conductivity is constant. **3** Heat generation is uniform.

Properties The conductivity and diffusivity are given to be k = 15 W/m·°C and $\alpha = 3.2\times10^{-6}$ m^2/s.



Analysis The nodal spacing is given to be Δx=Δx=l=0.015 m. The explicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{G}^i_{\text{element}} = \rho V_{\text{element}} C \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

The quantities h, T_∞, $\dot{g}$, and $\dot{q}_R$ do not change with time, and thus we do not need to use the superscript i for them. Also, the energy balance expressions can be simplified using the definitions of thermal diffusivity $\alpha = k/(\rho C)$ and the dimensionless mesh Fourier number $\tau = \alpha\Delta t / l^2$ where $\Delta x = \Delta y = l$. We note that all nodes are boundary nodes except node 5 that is an interior node. Therefore, we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 8 nodes are obtained as follows:

Node 1: $\dot{q}_L \dfrac{l}{2} + h\dfrac{l}{2}(T_\infty - T_1^i) + k\dfrac{l}{2}\dfrac{T_2^i - T_1^i}{l} + k\dfrac{l}{2}\dfrac{T_4^i - T_1^i}{l} + \dot{g}_0 \dfrac{l^2}{4} = \rho\dfrac{l^2}{4}C\dfrac{T_1^{i+1} - T_1^i}{\Delta t}$

Node 2: $hl(T_\infty - T_2^i) + k\dfrac{l}{2}\dfrac{T_1^i - T_2^i}{l} + k\dfrac{l}{2}\dfrac{T_3^i - T_2^i}{l} + kl\dfrac{T_5^i - T_2^i}{l} + \dot{g}_0\dfrac{l^2}{2} = \rho\dfrac{l^2}{2}C\dfrac{T_2^{i+1} - T_2^i}{\Delta t}$

Node 3: $hl(T_\infty - T_3^i) + k\dfrac{l}{2}\dfrac{T_2^i - T_3^i}{l} + k\dfrac{l}{2}\dfrac{T_6^i - T_3^i}{l} + \dot{g}_0\dfrac{l^2}{4} = \rho\dfrac{l^2}{4}C\dfrac{T_3^{i+1} - T_3^i}{\Delta t}$

(It can be rearranged as $T_3^{i+1} = \left(1 - 4\tau - 4\tau\dfrac{hl}{k}\right)T_3^i + 2\tau\left(T_4^i + T_6^i + 2\dfrac{hl}{k}T_\infty + \dfrac{\dot{g}_3 l^2}{2k}\right)$)

Node 4: $\dot{q}_L l + k\dfrac{l}{2}\dfrac{T_1^i - T_4^i}{l} + k\dfrac{l}{2}\dfrac{140 - T_4^i}{l} + kl\dfrac{T_5^i - T_4^i}{l} + \dot{g}_0\dfrac{l^2}{2} = \rho\dfrac{l^2}{2}C\dfrac{T_4^{i+1} - T_4^i}{\Delta t}$

Node 5 (interior): $T_5^{i+1} = (1 - 4\tau)T_5^i + \tau\left(T_2^i + T_4^i + T_6^i + 140 + \dfrac{\dot{g}_0 l^2}{k}\right)$

Node 6: $hl(T_\infty - T_6^i) + k\dfrac{l}{2}\dfrac{T_3^i - T_6^i}{l} + kl\dfrac{T_5^i - T_6^i}{l} + kl\dfrac{140 - T_6^i}{l} + k\dfrac{l}{2}\dfrac{T_7^i - T_6^i}{l} + \dot{g}_0\dfrac{3l^2}{4} = \rho\dfrac{3l^2}{4}C\dfrac{T_6^{i+1} - T_6^i}{\Delta t}$

Node 7: $hl(T_\infty - T_7^i) + k\dfrac{l}{2}\dfrac{T_6^i - T_7^i}{l} + k\dfrac{l}{2}\dfrac{T_8^i - T_7^i}{l} + kl\dfrac{140 - T_7^i}{l} + \dot{g}_0\dfrac{l^2}{2} = \rho\dfrac{l^2}{2}C\dfrac{T_7^{i+1} - T_7^i}{\Delta t}$

Node 8: $h\dfrac{l}{2}(T_\infty - T_8^i) + k\dfrac{l}{2}\dfrac{T_7^i - T_8^i}{l} + k\dfrac{l}{2}\dfrac{140 - T_8^i}{l} + \dot{g}_0\dfrac{l^2}{4} = \rho\dfrac{l^2}{4}C\dfrac{T_8^{i+1} - T_8^i}{\Delta t}$

where $\dot{g}_0 = 2\times10^7$ W/m^3, $\dot{q}_L = 8000$ W/m^2, l = 0.015 m, k =15 W/m·°C, h = 80 W/m^2·°C, and T_∞ =25°C.

The upper limit of the time step Δt is determined from the stability criteria that requires the coefficient of T_m^i in the T_m^{i+1} expression (the primary coefficient) be greater than or equal to zero for all nodes. The smallest primary coefficient in the 8 equations above is the coefficient of T_3^i in the T_3^{i+1} expression since it is exposed to most convection per unit volume (this can be verified), and thus the stability criteria for this problem can be expressed as

$$1-4\tau-4\tau\frac{hl}{k}\geq 0 \quad \rightarrow \quad \tau \leq \frac{1}{4(1+hl/k)} \quad \rightarrow \quad \Delta t \leq \frac{l^2}{4\alpha(1+hl/k)}$$

since $\tau = \alpha\Delta t/l^2$. Substituting the given quantities, the maximum allowable value of the time step is determined to be

$$\Delta t \leq \frac{(0.015\,\text{m})^2}{4(3.2\times10^{-6}\ \text{m}^2/\text{s})[1+(80\ \text{W/m}^2.°\text{C})(0.015\,\text{m})/(15\ \text{W/m}.°\text{C})]} = 16.3\,\text{s}$$

Therefore, any time step less than 16.3 s can be used to solve this problem. For convenience, we choose the time step to be Δt = 15 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha\Delta t}{l^2} = \frac{(3.2\times10^{-6}\ \text{m}^2/\text{s})(15\,\text{s})}{(0.015\,\text{m})^2} = 0.2133 \qquad (\text{for } \Delta t = 15\text{ s})$$

Using the specified initial condition as the solution at time t = 0 (for i = 0), sweeping through the 9 equations above will give the solution at intervals of 15 s. Using a computer, the solution at the upper corner node (node 3) is determined to be **441, 520**, and **529°C** at 2, 5, and 30 min, respectively. It can be shown that the steady state solution at node 3 is 531°C.

5-88 "!PROBLEM 5-88"

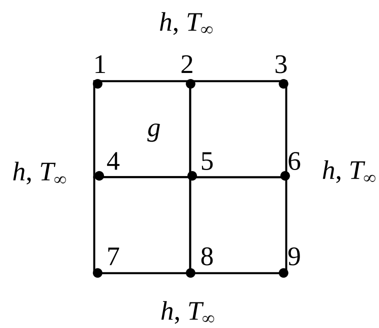
```
"GIVEN"
T_i=140 "[C]"
k=15 "[W/m-C]"
alpha=3.2E-6 "[m^2/s]"
g_dot=2E7 "[W/m^3]"
T_bottom=140 "[C]"
T_infinity=25 "[C]"
h=80 "[W/m^2-C]"
q_dot_L=8000 "[W/m^2]"
DELTAx=0.015 "[m]"
DELTAy=0.015 "[m]"
"time=120 [s], parameter to be varied"

"ANALYSIS"
l=DELTAx
DELTAt=15 "[s]"
tau=(alpha*DELTAt)/l^2
RhoC=k/alpha "RhoC=rho*C"
"The technique is to store the temperatures in the parametric table and recover them (as old
temperatures)
using the variable ROW.  The first row contains the initial values so Solve Table must begin
at row 2.
Use the DUPLICATE statement to reduce the number of equations that need to be typed.
Column 1
contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column
10 the Row."
Time=TableValue('Table 1',Row-1,#Time)+DELTAt
Duplicate i=1,8
    T_old[i]=TableValue('Table 1',Row-1,#T[i])
end
"Using the explicit finite difference approach, the eight equations for the eight unknown
temperatures are determined to be"
q_dot_L*l/2+h*l/2*(T_infinity-T_old[1])+k*l/2*(T_old[2]-T_old[1])/l+k*l/2*(T_old[4]-
T_old[1])/l+g_dot*l^2/4=RhoC*l^2/4*(T[1]-T_old[1])/DELTAt "Node 1"
h*l*(T_infinity-T_old[2])+k*l/2*(T_old[1]-T_old[2])/l+k*l/2*(T_old[3]-T_old[2])/l+k*l*(T_old[5]-
T_old[2])/l+g_dot*l^2/2=RhoC*l^2/2*(T[2]-T_old[2])/DELTAt "Node 2"
h*l*(T_infinity-T_old[3])+k*l/2*(T_old[2]-T_old[3])/l+k*l/2*(T_old[6]-
T_old[3])/l+g_dot*l^2/4=RhoC*l^2/4*(T[3]-T_old[3])/DELTAt "Node 3"
q_dot_L*l+k*l/2*(T_old[1]-T_old[4])/l+k*l/2*(T_bottom-T_old[4])/l+k*l*(T_old[5]-
T_old[4])/l+g_dot*l^2/2=RhoC*l^2/2*(T[4]-T_old[4])/DELTAt "Node 4"
T[5]=(1-4*tau)*T_old[5]+tau*(T_old[2]+T_old[4]+T_old[6]+T_bottom+g_dot*l^2/k) "Node 5"
h*l*(T_infinity-T_old[6])+k*l/2*(T_old[3]-T_old[6])/l+k*l*(T_old[5]-T_old[6])/l+k*l*(T_bottom-
T_old[6])/l+k*l/2*(T_old[7]-T_old[6])/l+g_dot*3/4*l^2=RhoC*3/4*l^2*(T[6]-T_old[6])/DELTAt
"Node 6"
h*l*(T_infinity-T_old[7])+k*l/2*(T_old[6]-T_old[7])/l+k*l/2*(T_old[8]-T_old[7])/l+k*l*(T_bottom-
T_old[7])/l+g_dot*l^2/2=RhoC*l^2/2*(T[7]-T_old[7])/DELTAt "Node 7"
h*l/2*(T_infinity-T_old[8])+k*l/2*(T_old[7]-T_old[8])/l+k*l/2*(T_bottom-
T_old[8])/l+g_dot*l^2/4=RhoC*l^2/4*(T[8]-T_old[8])/DELTAt "Node 8"
```

Time [s]	**T_1 [C]**	**T_2 [C]**	**T_3 [C]**	**T_4 [C]**	**T_5 [C]**	**T_6 [C]**	**T_7 [C]**	**T_8 [C]**	**Row**
0	140	140	140	140	140	140	140	140	1
15	203.5	200.1	196.1	207.4	204	201.4	200.1	200.1	2
30	265	259.7	252.4	258.2	253.7	243.7	232.7	232.5	3
45	319	312.7	300.3	299.9	293.5	275.7	252.4	250.1	4
60	365.5	357.4	340.3	334.6	326.4	300.7	265.2	260.4	5
75	404.6	394.9	373.2	363.6	353.5	320.6	274.1	267	6
90	437.4	426.1	400.3	387.8	375.9	336.7	280.8	271.6	7
105	464.7	451.9	422.5	407.9	394.5	349.9	286	275	8
120	487.4	473.3	440.9	424.5	409.8	360.7	290.1	277.5	9
135	506.2	491	456.1	438.4	422.5	369.6	293.4	279.6	10
…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…
1650	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	111
1665	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	112
1680	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	113
1695	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	114
1710	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	115
1725	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	116
1740	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	117
1755	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	118
1770	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	119
1785	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	120

5-89 A long solid bar is subjected to transient two-dimensional heat transfer. The centerline temperature of the bar after 10 min and after steady conditions are established are to be determined.

Assumptions **1** Heat transfer through the body is given to be transient and two-dimensional. **2** Heat is generated uniformly in the body. **3** The heat transfer coefficient also includes the radiation effects.

Properties The conductivity and diffusivity are given to be k = 28 W/m·°C and $\alpha = 12 \times 10^{-6}\ \text{m}^2/\text{s}$.



Analysis The nodal spacing is given to be Δx=Δx=l=0.1 m. The explicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{G}^{\text{i}}_{\text{element}} = \rho V_{\text{element}} C \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

The quantities h, T_∞, and $\dot{g}_0$ do not change with time, and thus we do not need to use the superscript i for them. The general explicit finite difference form of an interior node for transient two-dimensional heat conduction is expressed as

$$T^{i+1}_{\text{node}} = \tau(T^{\text{i}}_{\text{left}} + T^{\text{i}}_{\text{top}} + T^{\text{i}}_{\text{right}} + T^{\text{i}}_{\text{bottom}}) + (1-4\tau)T^{\text{i}}_{\text{node}} + \tau\frac{\dot{g}^{\text{i}}_{\text{node}} l^2}{k}$$

There is symmetry about the vertical, horizontal, and diagonal lines passing through the center. Therefore, $T_1 = T_3 = T_7 = T_9$ and $T_2 = T_4 = T_6 = T_8$, and T_1, T_2, and T_5 are the only 3 unknown nodal temperatures, and thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes. The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:

Node 1: $$hl(T_\infty - T_1^i) + k\frac{l}{2}\frac{T_2^i - T_1^i}{l} + k\frac{l}{2}\frac{T_4^i - T_1^i}{l} + \dot{g}_0\frac{l^2}{4} = \rho\frac{l^2}{4}C\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Node 2: $$h\frac{l}{2}(T_\infty - T_2^i) + k\frac{l}{2}\frac{T_1^i - T_2^i}{l} + k\frac{l}{2}\frac{T_5^i - T_2^i}{l} + \dot{g}_0\frac{l^2}{4} = \rho\frac{l^2}{4}C\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

Node 5 (interior): $$T_5^{i+1} = (1-4\tau)T_5^i + \tau\left(4T_2^i + \frac{\dot{g}_0 l^2}{k}\right)$$

where $\dot{g}_0 = 8 \times 10^5\ \text{W/m}^3$, l = 0.1 m, and k = 28 W/m·°C, h = 45 W/m^2·°C, and T_∞ =30°C.

The upper limit of the time step Δt is determined from the stability criteria that requires the coefficient of T_m^i in the T_m^{i+1} expression (the primary coefficient) be greater than or equal to zero for all nodes. The smallest primary coefficient in the 3 equations above is the coefficient of T_1^i in the T_1^{i+1} expression since it is exposed to most convection per unit volume (this can be verified), and thus the stability criteria for this problem can be expressed as

$$1 - 4\tau - 4\tau\frac{hl}{k} \geq 0 \quad \rightarrow \quad \tau \leq \frac{1}{4(1 + hl/k)} \quad \rightarrow \quad \Delta t \leq \frac{l^2}{4\alpha(1 + hl/k)}$$

since $\tau = \alpha\Delta t / l^2$. Substituting the given quantities, the maximum allowable value of the time step is determined to be

$$\Delta t \leq \frac{(0.1\,\text{m})^2}{4(12\times10^{-6}\ \text{m}^2/\text{s})[1 + (45\,\text{W/m}^2.°\text{C})(0.1\,\text{m})/(28\,\text{W/m.°C})]} = 179\,\text{s}$$

Therefore, any time step less than 179 s can be used to solve this problem. For convenience, we choose the time step to be Δt = 60 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha \Delta t}{l^2} = \frac{(12\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}{(0.1\,\text{m})^2} = 0.072 \qquad (\text{for}\quad \Delta t = 60\ \text{s})$$

Using the specified initial condition as the solution at time t = 0 (for i = 0), sweeping through the 3 equations above will give the solution at intervals of 1 min. Using a computer, the solution at the center node (node 5) is determined to be **217.2°C**, 302.8°C, 379.3°C, 447.7°C, 508.9°C, 612.4°C, 695.1°C, and 761.2°C at 10, 15, 20, 25, 30, 40, 50, and 60 min, respectively. Continuing in this manner, it is observed that steady conditions are reached in the medium after about 6 hours for which the temperature at the center node is **1023°C**.

5-90E A plain window glass initially at a uniform temperature is subjected to convection on both sides. The transient finite difference formulation of this problem is to be obtained, and it is to be determined how long it will take for the fog on the windows to clear up (i.e., for the inner surface temperature of the window glass to reach 54°F).

Assumptions **1** Heat transfer is one-dimensional since the window is large relative to its thickness. **2** Thermal conductivity is constant. **3** Radiation heat transfer is negligible.

Properties The conductivity and diffusivity are given to be k = 0.48 Btu/h.ft·°F and $\alpha = 4.2\times 10^{-6}\ \text{ft}^2/\text{s}$.

Analysis The nodal spacing is given to be Δx = 0.125 in. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.375/0.125+1 = 4. This problem involves 4 unknown nodal temperatures, and thus we need to have 4 equations. Nodes 2 and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{g}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau} \ \rightarrow\ T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i$$

since there is no heat generation. The finite difference equation for nodes 1 and 4 on the surfaces subjected to convection is obtained by applying an energy balance on the half volume element about the node, and taking the direction of all heat transfers to be towards the node under consideration:

Node 1 (convection): $$h_i(T_i - T_1^i) + k\frac{T_2^i - T_1^i}{\Delta x} = \rho\frac{\Delta x}{2}C\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

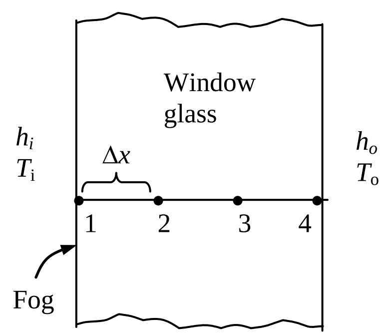
or $$T_1^{i+1} = \left(1 - 2\tau - 2\tau\frac{h_i \Delta x}{k}\right)T_1^i + 2\tau T_2^i + 2\tau\frac{h_i \Delta x}{k}T_i$$

Node 2 (interior): $$T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i$$

Node 3 (interior): $$T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i$$

Node 4 (convection): $$h_o(T_o - T_4^i) + k\frac{T_3^i - T_4^i}{\Delta x} = \rho\frac{\Delta x}{2}C\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

or $$T_4^{i+1} = \left(1 - 2\tau - 2\tau\frac{h_o \Delta x}{k}\right)T_4^i + 2\tau T_3^i + 2\tau\frac{h_o \Delta x}{k}T_o$$



where Δx = 0.125/12 ft , k = 0.48 Btu/h.ft·°F, h_i = 1.2 Btu/h.ft^2·°F, T_i =35+2*(t/60)°F (t in seconds), h_o = 2.6 Btu/h.ft^2·°F, and T_o =35°F. The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient of T_4^i is smaller in this case, and thus the stability criteria for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h\Delta x}{k} \geq 0 \quad \rightarrow \quad \tau \leq \frac{1}{2(1 + h\Delta x/k)} \quad \rightarrow \quad \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h\Delta x/k)}$$

since $\tau = \alpha\Delta t/\Delta x^2$. Substituting the given quantities, the maximum allowable time step becomes

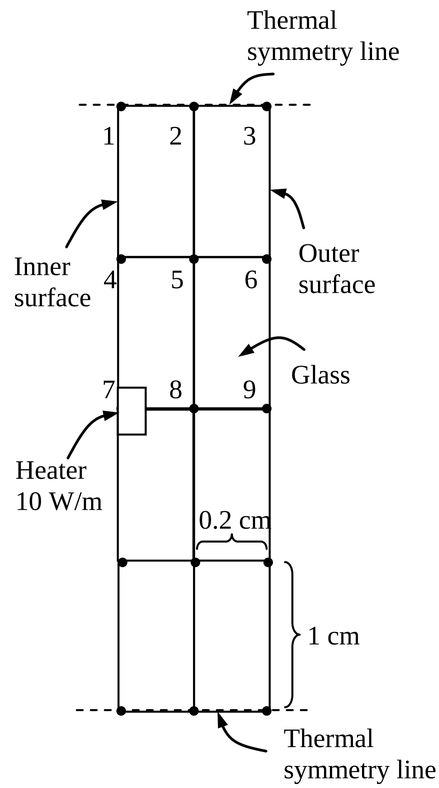
$$\Delta t \leq \frac{(0.125/12\,\text{ft})^2}{2(4.2\times 10^{-6}\ \text{ft}^2/\text{s})[1 + (2.6\,\text{Btu/h.ft}^2.°\text{F})(0.125/12\,\text{m})/(0.48\,\text{Btu/h.ft.°F})]} = 12.2\,\text{s}$$

Therefore, any time step less than 12.2 s can be used to solve this problem. For convenience, let us choose the time step to be Δt = 10 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(4.2\times 10^{-6}\ \text{ft}^2/\text{s})(10\ \text{s})}{(0.125/12\ \text{ft})^2} = 0.3871$$

Substituting this value of τ and other given quantities, the time needed for the inner surface temperature of the window glass to reach 54°F to avoid fogging is determined to be *never*. This is because steady conditions are reached in about 156 min, and the inner surface temperature at that time is determined to be 48.0°F. Therefore, the window will will be fogged at all times.

5-91 The formation of fog on the glass surfaces of a car is to be prevented by attaching electric resistance heaters to the inner surfaces. The temperature distribution throughout the glass 15 min after the strip heaters are turned on and also when steady conditions are reached are to be determined using the explicit method.



Assumptions **1** Heat transfer through the glass is given to be transient and two-dimensional. **2** Thermal conductivity is constant. **3** There is heat generation only at the inner surface, which will be treated as prescribed heat flux.

Properties The conductivity and diffusivity are given to be k = 0.84 W/m·°C and $\alpha = 0.39 \times 10^{-6}\ \text{m}^2/\text{s}$.

Analysis The nodal spacing is given to be Δx = 0.2 cm and Δy = 1 cm. The explicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{G}^i_{\text{element}} = \rho V_{\text{element}} C \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

We consider only 9 nodes because of symmetry. Note that we do not have a square mesh in this case, and thus we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 9 nodes are obtained as follows:

Node 1: $h_i \frac{\Delta y}{2}(T_i - T_1^i) + k\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_1^{i+1} - T_1^i}{\Delta t}$

Node 2: $k\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + k\Delta x\frac{T_5^i - T_2^i}{\Delta y} = \rho C\Delta x\frac{\Delta y}{2}\frac{T_2^{i+1} - T_2^i}{\Delta t}$

Node 3: $h_o \frac{\Delta y}{2}(T_o - T_3^i) + k\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_3^{i+1} - T_3^i}{\Delta t}$

Node 4: $h_i \Delta y(T_i - T_4^i) + k\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y} + k\Delta y\frac{T_5^i - T_4^i}{\Delta x} = \rho C\Delta y\frac{\Delta x}{2}\frac{T_4^{i+1} - T_4^i}{\Delta t}$

Node 5: $k\Delta y\frac{T_4^i - T_5^i}{\Delta x} + k\Delta y\frac{T_6^i - T_5^i}{\Delta x} + k\Delta x\frac{T_8^i - T_5^i}{\Delta y} + k\Delta x\frac{T_2^i - T_5^i}{\Delta y} = \rho C\Delta x\Delta y\frac{T_5^{i+1} - T_5^i}{\Delta t}$

Node 6: $h_o \Delta y(T_i - T_6^i) + k\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y} + k\Delta y\frac{T_5^i - T_6^i}{\Delta x} = \rho C\Delta y\frac{\Delta x}{2}\frac{T_6^{i+1} - T_6^i}{\Delta t}$

Node 7: $5\text{ W} + h_i \frac{\Delta y}{2}(T_i - T_7^i) + k\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^i - T_7^i}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_7^{i+1} - T_7^i}{\Delta t}$

Node 8: $k\frac{\Delta y}{2}\frac{T_7^i - T_8^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_9^i - T_8^i}{\Delta x} + k\Delta x\frac{T_5^i - T_8^i}{\Delta y} = \rho C\Delta x\frac{\Delta y}{2}\frac{T_8^{i+1} - T_8^i}{\Delta t}$

Node 9: $h_o \frac{\Delta y}{2}(T_o - T_9^i) + k\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^i - T_9^i}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_9^{i+1} - T_9^i}{\Delta t}$

where k = 0.84 W/m.°C, $\alpha = k/\rho C = 0.39\times10^{-6}\ \text{m}^2/\text{s}$, T_i = T_o = -3°C h_i = 6 W/m^2.°C, h_o = 20 W/m^2.°C, Δx = 0.002 m, and Δy = 0.01 m.

The upper limit of the time step Δt is determined from the stability criteria that requires the coefficient of T_m^i in the T_m^{i+1} expression (the primary coefficient) be greater than or equal to zero for all nodes. The smallest primary coefficient in the 9 equations above is the coefficient of T_9^i in the T_6^{i+1} expression since it is exposed to most convection per unit volume (this can be verified). The equation for node 6 can be rearranged as

$$T_6^{i+1} = \left[1 - 2\alpha\Delta t\left(\frac{h_o}{k\Delta x} + \frac{1}{\Delta y^2} + \frac{1}{\Delta x^2}\right)\right]T_6^i + 2\alpha\Delta t\left(\frac{h_o}{k\Delta x}T_0 + \frac{T_3^i + T_9^i}{\Delta y^2} + \frac{T_5^i}{\Delta x^2}\right)$$

Therefore, the stability criteria for this problem can be expressed as

$$1 - 2\alpha\Delta t\left(\frac{h_o}{k\Delta x} + \frac{1}{\Delta y^2} + \frac{1}{\Delta x^2}\right) \geq 0 \rightarrow \Delta t \leq \frac{1}{2\alpha\left(\frac{h_o}{k\Delta x} + \frac{1}{\Delta y^2} + \frac{1}{\Delta x^2}\right)}$$

Substituting the given quantities, the maximum allowable value of the time step is determined to be

or, $$\Delta t \leq \frac{1}{2\times(0.39\times10^{6}\ \text{m}^2/s)\left(\frac{20\ \text{W/m}^2\cdot°\text{C}}{(0.84\ \text{W/m}\cdot°\text{C})(0.002\,\text{m})} + \frac{1}{(0.002\,\text{m})^2} + \frac{1}{(0.01\,\text{m})^2}\right)} = 4.8\,\text{s}$$

Therefore, any time step less than 4.8 s can be used to solve this problem. For convenience, we choose the time step to be Δt = 4 s. Then the temperature distribution throughout the glass 15 min after the strip heaters are turned on and when steady conditions are reached are determined to be (from the EES solutions disk)

15 min: T_1 = -2.4°C, T_2 = -2.4°C, T_3 = -2.5°C, T_4 = -1.8°C, T_5 = -2.0°C,

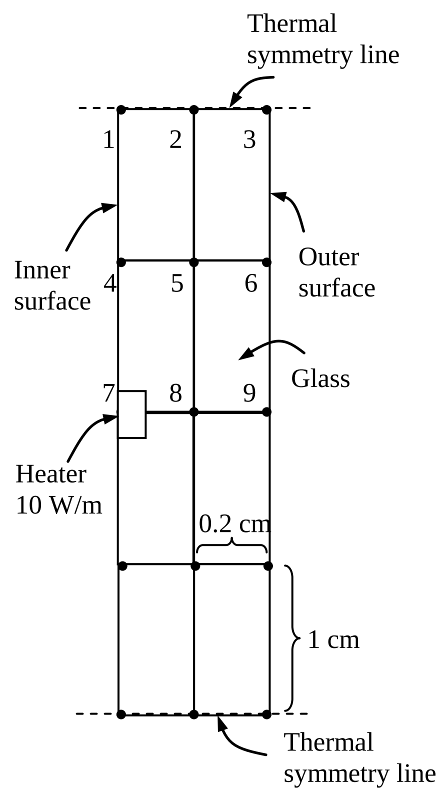
T_6 = -2.7°C, T_7 = 12.3°C, T_8 = 10.7°C, T_9 = 9.6°C

Steady-state: T_1 = -2.4°C, T_2 = -2.4°C, T_3 = -2.5°C, T_4 = -1.8°C, T_5 = -2.0°C,

T_6 = -2.7°C, T_7 = 12.3°C, T_8 = 10.7°C, T_9 = 9.6°C

Discussion Steady operating conditions are reached in about 8 min.

5-92 The formation of fog on the glass surfaces of a car is to be prevented by attaching electric resistance heaters to the inner surfaces. The temperature distribution throughout the glass 15 min after the strip heaters are turned on and also when steady conditions are reached are to be determined using the implicit method with a time step of Δt = 1 min.



Assumptions **1** Heat transfer through the glass is given to be transient and two-dimensional. **2** Thermal conductivity is constant. **3** There is heat generation only at the inner surface, which will be treated as prescribed heat flux.

Properties The conductivity and diffusivity are given to be k = 0.84 W/m·°C and $\alpha = 0.39 \times 10^{-6}\ \text{m}^2/\text{s}$.

Analysis The nodal spacing is given to be Δx = 0.2 cm and Δy = 1 cm. The implicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^{i+1} + \dot{G}^{i+1}_{\text{element}} = \rho V_{\text{element}} C \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

We consider only 9 nodes because of symmetry. Note that we do not have a square mesh in this case, and thus we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 9 nodes are obtained as follows:

Node 1: $h_i \frac{\Delta y}{2}(T_i - T_1^{i+1}) + k\frac{\Delta x}{2}\frac{T_4^{i+1} - T_1^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^{i+1} - T_1^{i+1}}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_1^{i+1} - T_1^i}{\Delta t}$

Node 2: $k\frac{\Delta y}{2}\frac{T_1^{i+1} - T_2^{i+1}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^{i+1} - T_2^{i+1}}{\Delta x} + k\Delta x\frac{T_5^{i+1} - T_2^{i+1}}{\Delta y} = \rho C \Delta x\frac{\Delta y}{2}\frac{T_2^{i+1} - T_2^i}{\Delta t}$

Node 3: $h_o \frac{\Delta y}{2}(T_o - T_3^{i+1}) + k\frac{\Delta x}{2}\frac{T_6^{i+1} - T_3^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^{i+1} - T_3^{i+1}}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_3^{i+1} - T_3^i}{\Delta t}$

N4: $h_i \Delta y(T_i - T_4^{i+1}) + k\frac{\Delta x}{2}\frac{T_1^{i+1} - T_4^{i+1}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^{i+1} - T_4^{i+1}}{\Delta y} + k\Delta y\frac{T_5^{i+1} - T_4^{i+1}}{\Delta x} = \rho C \Delta y\frac{\Delta x}{2}\frac{T_4^{i+1} - T_4^i}{\Delta t}$

Node 5: $k\Delta y\frac{T_4^{i+1} - T_5^{i+1}}{\Delta x} + k\Delta y\frac{T_6^{i+1} - T_5^{i+1}}{\Delta x} + k\Delta x\frac{T_8^{i+1} - T_5^{i+1}}{\Delta y} + k\Delta x\frac{T_2^{i+1} - T_5^{i+1}}{\Delta y} = \rho C \Delta x \Delta y\frac{T_5^{i+1} - T_5^i}{\Delta t}$

N6: $h_o \Delta y(T_i - T_6^{i+1}) + k\frac{\Delta x}{2}\frac{T_3^{i+1} - T_6^{i+1}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^{i+1} - T_6^{i+1}}{\Delta y} + k\Delta y\frac{T_5^{i+1} - T_6^{i+1}}{\Delta x} = \rho C \Delta y\frac{\Delta x}{2}\frac{T_6^{i+1} - T_6^i}{\Delta t}$

Node 7: $5\ \text{W} + h_i \frac{\Delta y}{2}(T_i - T_7^{i+1}) + k\frac{\Delta x}{2}\frac{T_4^{i+1} - T_7^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^{i+1} - T_7^{i+1}}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_7^{i+1} - T_7^i}{\Delta t}$

Node 8: $k\frac{\Delta y}{2}\frac{T_7^{i+1} - T_8^{i+1}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_9^{i+1} - T_8^{i+1}}{\Delta x} + k\Delta x\frac{T_5^{i+1} - T_8^{i+1}}{\Delta y} = \rho C \Delta x\frac{\Delta y}{2}\frac{T_8^{i+1} - T_8^i}{\Delta t}$

Node 9: $h_o \frac{\Delta y}{2}(T_o - T_9^{i+1}) + k\frac{\Delta x}{2}\frac{T_6^{i+1} - T_9^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^{i+1} - T_9^{i+1}}{\Delta x} = \rho C \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_9^{i+1} - T_9^i}{\Delta t}$

where k = 0.84 W/m.°C, $\alpha = k/\rho C = 0.39 \times 10^{-6}\ \text{m}^2/\text{s}$, $T_i = T_o$ = -3°C h_i = 6 W/m^2.°C, h_o = 20 W/m^2.°C, Δx = 0.002 m, and Δy = 0.01 m. Taking time step to be Δt = 1 min, the temperature distribution throughout the glass 15 min after the strip heaters are turned on and when steady conditions are reached are determined to be (from the EES solutions disk)

15 min: T_1 = -2.4°C, T_2 = -2.4°C, T_3 = -2.5°C, T_4 = -1.8°C, T_5 = -2.0°C,

T_6 = -2.7°C, T_7 = 12.3°C, T_8 = 10.7°C, T_9 = 9.6°C

Steady-state: T_1 = -2.4°C, T_2 = -2.4°C, T_3 = -2.5°C, T_4 = -1.8°C, T_5 = -2.0°C,

T_6 = -2.7°C, T_7 = 12.3°C, T_8 = 10.7°C, T_9 = 9.6°C

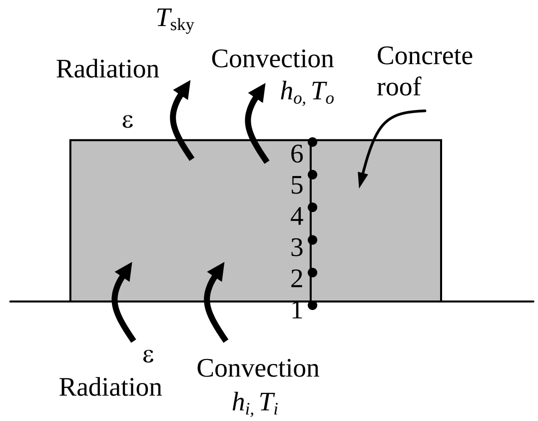
Discussion Steady operating conditions are reached in about 8 min.

5-93 The roof of a house initially at a uniform temperature is subjected to convection and radiation on both sides. The temperatures of the inner and outer surfaces of the roof at 6 am in the morning as well as the average rate of heat transfer through the roof during that night are to be determined.

Assumptions **1** Heat transfer is one-dimensional. **2** Thermal properties, heat transfer coefficients, and the indoor and outdoor temperatures are constant. **3** Radiation heat transfer is significant.

Properties The conductivity and diffusivity are given to be k = 1.4 W/m.°C and $\alpha = 0.69\times10^{-6}\ \text{m}^2/\text{s}$. The emissivity of both surfaces of the concrete roof is 0.9.

Analysis The nodal spacing is given to be Δx = 0.03 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.15/0.03+1 = 6. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations. Nodes 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as



$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{g}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$\rightarrow\ T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{g}_m^i \Delta x^2}{k}$$

The finite difference equations for nodes 1 and 6 subjected to convection and radiation are obtained from an energy balance by taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}:\quad h_i(T_i - T_1^i) + k\frac{T_2^i - T_1^i}{\Delta x} + \varepsilon\sigma\left[T_{wall}^4 - (T_1^i + 273)^4\right] = \rho\frac{\Delta x}{2}C\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

$$\text{Node 2 (interior)}:\quad T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i$$

$$\text{Node 3 (interior)}:\quad T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i$$

$$\text{Node 4 (interior)}:\quad T_4^{i+1} = \tau(T_3^i + T_5^i) + (1-2\tau)T_4^i$$

$$\text{Node 5 (interior)}:\quad T_5^{i+1} = \tau(T_4^i + T_6^i) + (1-2\tau)T_5^i$$

$$\text{Node 6 (convection)}:\quad h_o(T_0 - T_6^i) + k\frac{T_5^i - T_6^i}{\Delta x} + \varepsilon\sigma\left[T_{sky}^4 - (T_6^i + 273)^4\right] = \rho\frac{\Delta x}{2}C\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

where k = 1.4 W/m.°C, $\alpha = k/\rho C = 0.69\times10^{-6}\ \text{m}^2/\text{s}$, T_i = 20°C, T_{wall} = 293 K, T_o = 6°C, T_{sky} =260 K, h_i = 5 W/m^2.°C, h_o = 12 W/m^2.°C, Δx = 0.03 m, and Δt = 5 min. Also, the mesh Fourier number is

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(0.69\times10^{-6}\ \text{m}^2/\text{s})(300\ \text{s})}{(0.03\ \text{m})^2} = 0.230$$

Substituting this value of τ and other given quantities, the inner and outer surface temperatures of the roof after 12×(60/5) = 144 time steps (12 h) are determined to be $\boldsymbol{T_1}$ **= 10.3°C and** $\boldsymbol{T_6}$ **= -0.97°C.**

(*b*) The average temperature of the inner surface of the roof can be taken to be

$$T_{1,ave} = \frac{T_{1@6\,\text{PM}} + T_{1@6\,\text{AM}}}{2} = \frac{18+10.3}{2} = 14.15°\text{C}$$

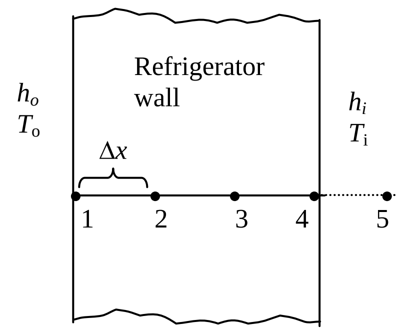
Then the average rate of heat loss through the roof that night becomes

$$\begin{aligned}\dot{Q}_{ave} &= h_i A_s(T_i - T_{1,ave}) + \varepsilon\sigma A_s\left[T_{wall}^4 - (T_1^i + 273)^4\right]\\ &= (5\ \text{W/m}^2\cdot°\text{C})(20\times20\ \text{m}^2)(20\text{-}14.15)°\text{C} + 0.9(20\times20\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(293\text{K})^4 - (14.15+273\text{K})^4]\\ &= \mathbf{23{,}360\ W}\end{aligned}$$

5-94 A refrigerator whose walls are constructed of 3-cm thick urethane insulation malfunctions, and stops running for 6 h. The temperature inside the refrigerator at the end of this 6 h period is to be determined.

Assumptions **1** Heat transfer is one-dimensional since the walls are large relative to their thickness. **2** Thermal properties, heat transfer coefficients, and the outdoor temperature are constant. **3** Radiation heat transfer is negligible. **4** The temperature of the contents of the refrigerator, including the air inside, rises uniformly during this period. **5** The local atmospheric pressure is 1 atm. **6** The space occupied by food and the corner effects are negligible. **7** Heat transfer through the bottom surface of the refrigerator is negligible.

Properties The conductivity and diffusivity are given to be k = 1.4 W/m.°C and $\alpha = 0.69\times10^{-6}\ \text{m}^2/\text{s}$. The average specific heat of food items is given to be 3.6 kJ/kg.°C. The specific heat and density of air at 1 atm and 3°C are C_p = 1.004 kJ/kg.°C and ρ = 1.29 kg/m^3 (Table A-15.



Analysis The nodal spacing is given to be Δx = 0.01 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.03/0.01+1 = 4. This problem involves 4 unknown nodal temperatures, and thus we need to have 4 equations. Nodes 2 and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{g}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau} \ \rightarrow\ T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{g}_m^i \Delta x^2}{k}$$

The finite difference equations for nodes 1 and 4 subjected to convection and radiation are obtained from an energy balance by taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}:\quad h_o(T_0 - T_1^i) + k\frac{T_2^i - T_1^i}{\Delta x} = \rho\frac{\Delta x}{2}C\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

$$\text{Node 2 (interior)}:\quad T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i$$

$$\text{Node 3 (interior)}:\quad T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i$$

$$\text{Node 4 (convection)}:\quad h_i(T_5^i - T_4^i) + k\frac{T_3^i - T_4^i}{\Delta x} = \rho\frac{\Delta x}{2}C\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

where k = 0.026 W/m.°C, $\alpha = k/\rho C = 0.36\times10^{-6}\ \text{m}^2/\text{s}$, $T_5 = T_i$ = 3°C (initially), T_o = 25°C, h_i = 6 W/m^2.°C, h_o = 9 W/m^2.°C, Δx = 0.01 m, and Δt = 1 min. Also, the mesh Fourier number is

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(0.39\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{s})}{(0.01\ \text{m})^2} = 0.216$$

The volume of the refrigerator cavity and the mass of air inside are

$$V = (1.80-0.03)(0.8-0.03)(0.7-0.03) = 0.913\text{m}^3$$

$$m_{air} = \rho V = (1.29\ \text{kg/m}^3)(0.824\ \text{m}^3) = 1.063\ \text{kg}$$

Energy balance for the air space of the refrigerator can be expressed as

$$\text{Node 5 (refrig. air)}:\quad h_i A_i(T_4^i - T_5^i) = (mC\Delta T)_{air} + (mC\Delta T)_{food}$$

or

$$h_i A_i(T_4^i - T_5^i) = \left[(mC)_{air} + (mC)_{food}\right]\frac{T_5^{i+1} - T_5^i}{\Delta t}$$

where $$A_i = 2(1.77\times0.77) + 2(1.77\times0.67) + (0.77\times0.67) = 5.6135\text{m}^2$$

Substituting, temperatures of the refrigerated space after 6×60 = 360 time steps (6 h) is determined to be

$\boldsymbol{T_{in} = T_5}$ **=19.6°C.**

5-95 "!PROBLEM 5-95"

```
"GIVEN"
t_ins=0.03 "[m]"
k=0.026 "[W/m-C]"
alpha=0.36E-6 "[m^2/s]"
T_i=3 "[C]"
h_i=6 "[W/m^2-C]"
h_o=9 "[W/m^2-C]"
T_infinity=25 "[C]"
m_food=15 "[kg]"
C_food=3600 "[J/kg-C]"
DELTAx=0.01 "[m]"
DELTAt=60 "[s]"
"time=6*3600 [s], parameter to be varied"

"PROPERTIES"
rho_air=density(air, T=T_i, P=101.3)
C_air=CP(air, T=T_i)*Convert(kJ/kg-C, J/kg-C)

"ANALYSIS"
M=t_ins/DELTAx+1 "Number of nodes"
tau=(alpha*DELTAt)/DELTAx^2
RhoC=k/alpha "RhoC=rho*C"

"The technique is to store the temperatures in the parametric table and recover them (as old
temperatures)
using the variable ROW.  The first row contains the initial values so Solve Table must begin
at row 2.
Use the DUPLICATE statement to reduce the number of equations that need to be typed.
Column 1
contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 7
the Row."
Time=TableValue('Table 1',Row-1,#Time)+DELTAt
Duplicate i=1,5
    T_old[i]=TableValue('Table 1',Row-1,#T[i])
end

"Using the explicit finite difference approach, the six equations for the six unknown
temperatures are determined to be"
h_o*(T_infinity-T_old[1])+k*(T_old[2]-T_old[1])/DELTAx=RhoC*DELTAx/2*(T[1]-
T_old[1])/DELTAt "Node 1, convection"
T[2]=tau*(T_old[1]+T_old[3])+(1-2*tau)*T_old[2] "Node 2"
T[3]=tau*(T_old[2]+T_old[4])+(1-2*tau)*T_old[3] "Node 3"
h_i*(T_old[5]-T_old[4])+k*(T_old[3]-T_old[4])/DELTAx=RhoC*DELTAx/2*(T[4]-
T_old[4])/DELTAt "Node 4, convection"
h_i*A_i*(T_old[4]-T_old[5])=m_air*C_air*(T[5]-T_old[5])/DELTAt+m_food*C_food*(T[5]-
T_old[5])/DELTAt "Node 5, refrig. air"

A_i=2*(1.8-0.03)*(0.8-0.03)+2*(1.8-0.03)*(0.7-0.03)+(0.8-0.03)*(0.7-0.03)
m_air=rho_air*V_air
V_air=(1.8-0.03)*(0.8-0.03)*(0.7-0.03)
```

Time [s]	**T_1 [C]**	**T_2 [C]**	**T_3 [C]**	**T_4 [C]**	**T_5 [C]**	**Row**
0	3	3	3	3	3	1
60	35.9	3	3	3	3	2
120	5.389	10.11	3	3	3	3
180	36.75	7.552	4.535	3	3	4
240	6.563	13.21	4.855	3.663	3	5
300	37	9.968	6.402	3.517	3.024	6
360	7.374	15.04	6.549	4.272	3.042	7
420	37.04	11.55	7.891	4.03	3.087	8
480	8.021	16.27	7.847	4.758	3.122	9
540	36.97	12.67	8.998	4.461	3.182	10
…	…	…	…	…	…	…
…	…	…	…	…	…	…
35460	24.85	24.23	23.65	23.09	22.86	592
35520	24.81	24.24	23.65	23.1	22.87	593
35580	24.85	24.23	23.66	23.11	22.88	594
35640	24.81	24.24	23.67	23.12	22.88	595
35700	24.85	24.24	23.67	23.12	22.89	596
35760	24.81	24.25	23.68	23.13	22.9	597
35820	24.85	24.25	23.68	23.14	22.91	598
35880	24.81	24.26	23.69	23.15	22.92	599
35940	24.85	24.25	23.69	23.15	22.93	600
36000	24.82	24.26	23.7	23.16	22.94	601