Chapter 6
FUNDAMENTALS OF CONVECTION

Physical Mechanisms of Forced Convection

6-1C In forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect that manifests itself as the rise of the warmer fluid and the fall of the cooler fluid. The convection caused by winds is natural convection for the earth, but it is forced convection for bodies subjected to the winds since for the body it makes no difference whether the air motion is caused by a fan or by the winds.

6-2C If the fluid is forced to flow over a surface, it is called external forced convection. If it is forced to flow in a tube, it is called internal forced convection. A heat transfer system can involve both internal and external convection simultaneously. Example: A pipe transporting a fluid in a windy area.

6-3C The convection heat transfer coefficient will usually be higher in forced convection since heat transfer coefficient depends on the fluid velocity, and forced convection involves higher fluid velocities.

6-4C The potato will normally cool faster by blowing warm air to it despite the smaller temperature difference in this case since the fluid motion caused by blowing enhances the heat transfer coefficient considerably.

6-5C Nusselt number is the dimensionless convection heat transfer coefficient, and it represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. It is defined as $Nu = \frac{hL}{k}$ where *L* is the characteristic length of the surface and *k* is the thermal conductivity of the fluid.

6-6C Heat transfer through a fluid is conduction in the absence of bulk fluid motion, and convection in the presence of it. The rate of heat transfer is higher in convection because of fluid motion. The value of the convection heat transfer coefficient depends on the fluid motion as well as the fluid properties. Thermal conductivity is a fluid property, and its value does not depend on the flow.

6-7C A fluid flow during which the density of the fluid remains nearly constant is called *incompressible flow*. A fluid whose density is practically independent of pressure (such as a liquid) is called an incompressible fluid. The flow of compressible fluid (such as air) is not necessarily compressible since the density of a compressible fluid may still remain constant during flow.

6-8 Heat transfer coefficients at different air velocities are given during air cooling of potatoes. The initial rate of heat transfer from a potato and the temperature gradient at the potato surface are to be determined.

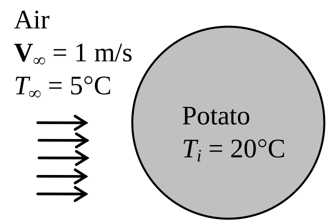
Assumptions **1** Steady operating conditions exist. **2** Potato is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface.

Properties The thermal conductivity of the potato is given to be k = 0.49 W/m.°C.

Analysis The initial rate of heat transfer from a potato is

$$A_s = \pi D^2 = \pi(0.10\,\text{m})^2 = 0.03142\,\text{m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (19.1\,\text{W/m}^2.°\text{C})(0.03142\,\text{m}^2)(20-5)°\text{C} = \mathbf{9.0\,W}$$



where the heat transfer coefficient is obtained from the table at 1 m/s velocity. The initial value of the temperature gradient at the potato surface is

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{(19.1\,\text{W/m}^2.°\text{C})(20-5)°\text{C}}{(0.49\,\text{W/m.°C})} = \mathbf{-585\,°C/m}$$

6-9 The rate of heat loss from an average man walking in still air is to be determined at different walking velocities.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is constant over the entire surface.

Analysis The convection heat transfer coefficients and the rate of heat losses at different walking velocities are

(*a*) $h = 8.6\mathbf{V}^{0.53} = 8.6(0.5\,\text{m/s})^{0.53} = 5.956\,\text{W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.956\,\text{W/m}^2.°\text{C})(1.8\,\text{m}^2)(30-10)°\text{C} = \mathbf{214.4\,W}$$

(*b*) $h = 8.6\mathbf{V}^{0.53} = 8.6(1.0\,\text{m/s})^{0.53} = 8.60\,\text{W/m}^2.°\text{C}$

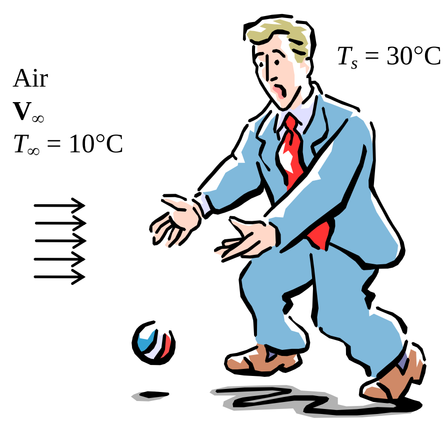
$$\dot{Q} = hA_s(T_s - T_\infty) = (8.60\,\text{W/m}^2.°\text{C})(1.8\,\text{m}^2)(30-10)°\text{C} = \mathbf{309.6\,W}$$

(*c*) $h = 8.6\mathbf{V}^{0.53} = 8.6(1.5\,\text{m/s})^{0.53} = 10.66\,\text{W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (10.66\,\text{W/m}^2.°\text{C})(1.8\,\text{m}^2)(30-10)°\text{C} = \mathbf{383.8\,W}$$

(*d*) $h = 8.6\mathbf{V}^{0.53} = 8.6(2.0\,\text{m/s})^{0.53} = 12.42\,\text{W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (12.42\,\text{W/m}^2.°\text{C})(1.8\,\text{m}^2)(30-10)°\text{C} = \mathbf{447.0\,W}$$



6-10 The rate of heat loss from an average man walking in windy air is to be determined at different wind velocities.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is constant over the entire surface.

Analysis The convection heat transfer coefficients and the rate of heat losses at different wind velocities are

(*a*) $h = 14.8\mathbf{V}^{0.53} = 14.8(0.5\text{ m/s})^{0.69} = 9.174\text{ W/m}^2.°\text{C}$

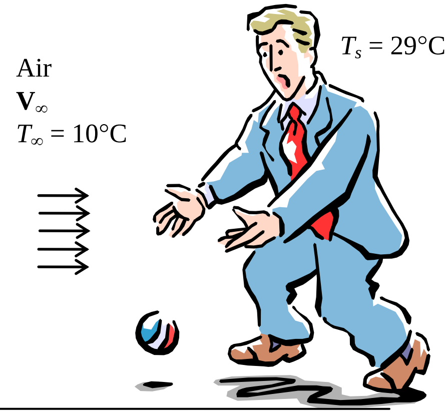
$$\dot{Q} = hA_s(T_s - T_\infty) = (9.174\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{296.3\ W}$$

(*b*) $h = 14.8\mathbf{V}^{0.53} = 14.8(1.0\text{ m/s})^{0.69} = 14.8\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (14.8\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{478.0\ W}$$

(*c*) $h = 14.8\mathbf{V}^{0.53} = 14.8(1.5\text{ m/s})^{0.69} = 19.58\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (19.58\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{632.4\ W}$$



6-11 The expression for the heat transfer coefficient for air cooling of some fruits is given. The initial rate of heat transfer from an orange, the temperature gradient at the orange surface, and the value of the Nusselt number are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Orange is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Properties of water is used for orange.

Properties The thermal conductivity of the orange is given to be k = 0.50 W/m.°C. The thermal conductivity and the kinematic viscosity of air at the film temperature of $(T_s + T_\infty)/2 = (15+5)/2 = 10°\text{C}$ are (Table A-15)

$$k = 0.02439\text{ W/m.°C}, \qquad \upsilon = 1.426\times10^{-5}\text{ m}^2\text{/s}$$

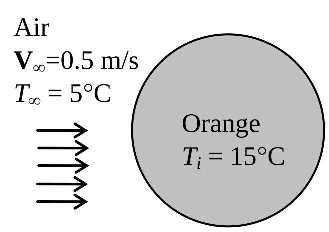
Analysis (*a*) The Reynolds number, the heat transfer coefficient, and the initial rate of heat transfer from an orange are

$$A_s = \pi D^2 = \pi(0.07\text{ m})^2 = 0.01539\text{ m}^2$$

$$\text{Re} = \frac{\mathbf{V}_\infty D}{\upsilon} = \frac{(0.5\text{ m/s})(0.07\text{ m})}{1.426\times10^{-5}\text{ m}^2\text{/s}} = 2454$$

$$h = \frac{5.05k_{air}\text{ Re}^{1/3}}{D} = \frac{5.05(0.02439\text{ W/m.°C})(2454)^{1/3}}{0.07\text{ m}} = 23.73\text{ W/m}^2.°\text{C}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (23.73\text{ W/m}^2.°\text{C})(0.01539\text{m}^2)(15-5)°\text{C} = \mathbf{3.65\ W}$$



(*b*) The temperature gradient at the orange surface is determined from

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{(23.73\text{ W/m}^2.°\text{C})(15-5)°\text{C}}{(0.50\text{ W/m.°C})} = \mathbf{-475\ °C/m}$$

(*c*) The Nusselt number is $\text{Re} = \dfrac{hD}{k} = \dfrac{(23.73\text{ W/m}^2.°\text{C})(0.07\text{ m})}{0.02439\text{ W/m.°C}} = \mathbf{68.11}$

Velocity and Thermal Boundary Layers

6-12C Viscosity is a measure of the "stickiness" or "resistance to deformation" of a fluid. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. Liquids have higher dynamic viscosities than gases.

6-13C The fluids whose shear stress is proportional to the velocity gradient are called *Newtonian fluids*. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids.

6-14C A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition*, and it is due to the viscosity of the fluid.

6-15C For the same cruising speed, the submarine will consume much less power in air than it does in water because of the much lower viscosity of air relative to water.

6-16C (*a*) The dynamic viscosity of liquids decreases with temperature. (*b*) The dynamic viscosity of gases increases with temperature.

6-17C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

6-18C The Prandtl number $\mathrm{Pr} = \nu / \alpha$ is a measure of the relative magnitudes of the diffusivity of momentum (and thus the development of the velocity boundary layer) and the diffusivity of heat (and thus the development of the thermal boundary layer). The Pr is a fluid property, and thus its value is independent of the type of flow and flow geometry. The Pr changes with temperature, but not pressure.

6-19C A thermal boundary layer will not develop in flow over a surface if both the fluid and the surface are at the same temperature since there will be no heat transfer in that case.

Laminar and Turbulent Flows

6-20C A fluid motion is laminar when it involves smooth streamlines and highly ordered motion of molecules, and turbulent when it involves velocity fluctuations and highly disordered motion. The heat transfer coefficient is higher in turbulent flow.

6-21C Reynolds number is the ratio of the inertial forces to viscous forces, and it serves as a criteria for determining the flow regime. For flow over a plate of length L it is defined as Re = $\mathbf{V}L/\upsilon$ where **V** is flow velocity and υ is the kinematic viscosity of the fluid.

6-22C The friction coefficient represents the resistance to fluid flow over a flat plate. It is proportional to the drag force acting on the plate. The drag coefficient for a flat surface is equivalent to the mean friction coefficient.

6-23C In turbulent flow, it is the *turbulent eddies* due to enhanced mixing that cause the friction factor to be larger.

6-24C Turbulent viscosity μ_t is caused by turbulent eddies, and it accounts for momentum transport by turbulent eddies. It is expressed as $\tau_t = -\rho \overline{u'v'} = \mu_t \dfrac{\partial \bar{u}}{\partial y}$ where $\bar{u}$ is the mean value of velocity in the flow direction and u' and u' are the fluctuating components of velocity.

6-25C Turbulent thermal conductivity k_t is caused by turbulent eddies, and it accounts for thermal energy transport by turbulent eddies. It is expressed as $\dot{q}_t = \rho C_p \overline{v'T'} = -k_t \dfrac{\partial \overline{T}}{\partial y}$ where T' is the eddy temperature relative to the mean value, and $\dot{q}_t = \rho C_p v'T'$ the rate of thermal energy transport by turbulent eddies.

Convection Equations and Similarity Solutions

6-26C A curved surface can be treated as a flat surface if there is no flow separation and the curvature effects are negligible.

6-27C The continuity equation for steady two-dimensional flow is expressed as $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$. When multiplied by density, the first and the second terms represent net mass fluxes in the x and y directions, respectively.

6-28C *Steady* simply means no change with time at a specified location (and thus $\partial u / \partial t = 0$), but the value of a quantity may change from one location to another (and thus $\partial u / \partial x$ and $\partial u / \partial y$ may be different from zero). Even in steady flow and thus constant mass flow rate, a fluid may accelerate. In the case of a water nozzle, for example, the velocity of water remains constant at a specified point, but it changes from inlet to the exit (water accelerates along the nozzle).

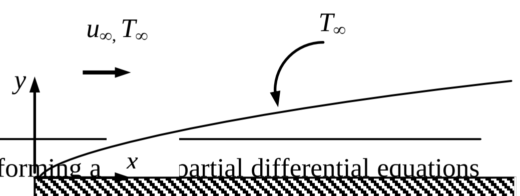
6-29C In a boundary layer during steady two-dimensional flow, the velocity component in the flow direction is much larger than that in the normal direction, and thus *u* >> *v*, and $\partial v / \partial x$ and $\partial v / \partial y$ are negligible. Also, *u* varies greatly with *y* in the normal direction from zero at the wall surface to nearly the free-stream value across the relatively thin boundary layer, while the variation of *u* with *x* along the flow is typically small. Therefore, $\partial u / \partial y >> \partial u / \partial x$. Similarly, if the fluid and the wall are at different temperatures and the fluid is heated or cooled during flow, heat conduction will occur primarily in the direction normal to the surface, and thus $\partial T / \partial y >> \partial T / \partial x$. That is, the velocity and temperature gradients normal to the surface are much greater than those along the surface. These simplifications are known as the **boundary layer approximations**.

6-30C For flows with low velocity and for fluids with low viscosity the viscous dissipation term in the energy equation is likely to be negligible.

6-31C For steady two-dimensional flow over an isothermal flat plate in the *x*-direction, the boundary conditions for the velocity components *u* and *v*, and the temperature *T* at the plate surface and at the edge of the boundary layer are expressed as follows:

At $y = 0$: $u(x, 0) = 0$, $v(x, 0) = 0$, $T(x, 0) = T_s$

As $y \to \infty$: $u(x, \infty) = u_\infty$, $T(x, \infty) = T_\infty$



6-32C An independent variable that makes it possible to transforming a set of partial differential equations into a single ordinary differential equation is called a **similarity variable**. A similarity solution is likely to exist for a set of partial differential equations if there is a function that remains unchanged (such as the non-dimensional velocity profile on a flat plate).

6-33C During steady, laminar, two-dimensional flow over an isothermal plate, the thickness of the velocity boundary layer (*a*) increase with distance from the leading edge, (*b*) decrease with free-stream velocity, and (*c*) and increase with kinematic viscosity

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6-34C During steady, laminar, two-dimensional flow over an isothermal plate, the wall shear stress decreases with distance from the leading edge

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6-35C A major advantage of nondimensionalizing the convection equations is the significant reduction in the number of parameters [the original problem involves 6 parameters (L, $\mathcal{V}$, T_∞, T_s, ν, α), but the nondimensionalized problem involves just 2 parameters (Re_L and Pr)]. Nondimensionalization also results in similarity parameters (such as Reynolds and Prandtl numbers) that enable us to group the results of a large number of experiments and to report them conveniently in terms of such parameters.

6-36C For steady, laminar, two-dimensional, incompressible flow with constant properties and a Prandtl number of unity and a given geometry, yes, it is correct to say that both the average friction and heat transfer coefficients depend on the Reynolds number only since $C_f = f_4(\mathrm{Re}_L)$ and $\mathrm{Nu} = g_3(\mathrm{Re}_L, \mathrm{Pr})$ from non-dimensionalized momentum and energy equations.

6-37 Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the heat flux are to be determined.

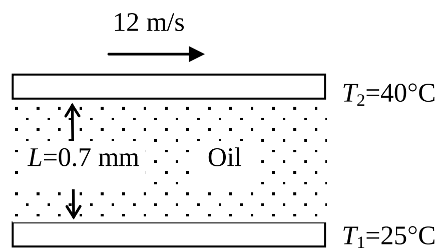
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in z direction.

Properties The properties of oil at the average temperature of (40+15)/2 = 27.5°C are (Table A-13):

k = 0.145 W/m-K and μ = 0.580 kg/m-s = 0.580 N-s/m^2

Analysis (a) We take the x-*axis* to be the flow direction, and y to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$



Therefore, the x-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, v = 0, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the x-momentum equation (Eq. 6-28) reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0)$ = 0 and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}\mathrm{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and v = 0. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{\mathrm{V}}{L}\right)^2$$

since $\partial u/\partial y = \mathrm{V}/L$. Dividing both sides by k and integrating twice give

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}\mathrm{V}\right)^2+C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives the temperature distribution to be

$$T(y)=\frac{T_2-T_1}{L}y+T_1+\frac{\mu \mathrm{V}^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

(b) The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu \mathrm{V}^2}{2kL}\left(1-2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting dT/dy = 0 and solving for y,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu \mathrm{V}^2}{2kL}\left(1-2\frac{y}{L}\right)=0 \longrightarrow y=L\left(k\frac{T_2-T_1}{\mu \mathrm{V}^2}+\frac{1}{2}\right)$$

The maximum temperature is the value of temperature at this y, whose numeric value is

$$y = L\left(k\frac{T_2 - T_1}{\mu V^2} + \frac{1}{2}\right) = (0.0007\,\text{m})\left[(0.145\,\text{W/m.°C})\frac{(40-15)\text{°C}}{(0.580\,\text{N.s/m}^2)(12\,\text{m/s})^2} + \frac{1}{2}\right]$$

$$= 0.0003804\text{m} = \mathbf{0.3804mm}$$

Then

$$T_{\max} = T(0.0003804) = \frac{T_2 - T_1}{L}y + T_1 + \frac{\mu V^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$= \frac{(40-15)\text{°C}}{0.0007\,\text{m}}(0.0003804\text{m}) + 15\text{°C} + \frac{(0.58\,\text{N}\cdot\text{s/m}^2)(12\,\text{m/s})^2}{2(0.145\text{W/m}\cdot\text{°C})}\left(\frac{0.0003804\text{m}}{0.0007\,\text{m}} - \frac{(0.0003804\text{m})^2}{(0.0007\,\text{m})^2}\right)$$

$$= \mathbf{100.0\,°C}$$

(c) Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-0) = -k\frac{T_2 - T_1}{L} - \frac{\mu V^2}{2L}$$

$$= -(0.145\,\text{W/m.°C})\frac{(40-15)\text{°C}}{0.0007\,\text{m}} - \frac{(0.58\,\text{N}\cdot\text{s/m}^2)(12\,\text{m/s})^2}{2(0.0007\text{m})}\left(\frac{1\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-6.48\times10^4\ W/m^2}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-2) = -k\frac{T_2 - T_1}{L} + \frac{\mu V^2}{2L}$$

$$= -(0.145\,\text{W/m.°C})\frac{(40-15)\text{°C}}{0.0007\,\text{m}} + \frac{(0.58\,\text{N}\cdot\text{s/m}^2)(12\,\text{m/s})^2}{2(0.0007\text{m})}\left(\frac{1\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{5.45\times10^4\ W/m^2}$$

Discussion A temperature rise of about 72.5°C confirms our suspicion that viscous dissipation is very significant. Calculations are done using oil properties at 27.5°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of about 64°C to improve accuracy.

6-38 Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the heat flux are to be determined.

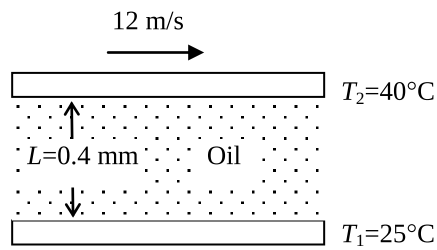
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in *z* direction.

Properties The properties of oil at the average temperature of (40+15)/2 = 27.5°C are (Table A-13):

k = 0.145 W/m-K and μ = 0.580 kg/m-s = 0.580 N-s/m^2

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation (Eq. 6-21) reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u=u(y)$$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, v = 0, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0)$ = 0 and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}\mathcal{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and v = 0. Then the energy equation with dissipation reduces to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2 T}{dy^2}=-\mu\left(\frac{\mathcal{V}}{L}\right)^2$$

since $\partial u/\partial y = \mathcal{V}/L$. Dividing both sides by *k* and integrating twice give

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2+C_3 y+C_4$$

Applying the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives the temperature distribution to be

$$T(y)=\frac{T_2-T_1}{L}y+T_1+\frac{\mu\mathcal{V}^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

(*b*) The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu\mathcal{V}^2}{2kL}\left(1-2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting dT/dy = 0 and solving for *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu\mathcal{V}^2}{2kL}\left(1-2\frac{y}{L}\right)=0 \longrightarrow y=L\left(k\frac{T_2-T_1}{\mu\mathcal{V}^2}+\frac{1}{2}\right)$$

The maximum temperature is the value of temperature at this *y*, whose numeric value is

$$y = L\left(k\frac{T_2 - T_1}{\mu V^2} + \frac{1}{2}\right) = (0.0004\,\text{m})\left[(0.145\,\text{W/m.°C})\frac{(40-15)\text{°C}}{(0.580\,\text{N.s/m}^2)(12\,\text{m/s})^2} + \frac{1}{2}\right]$$

$$= 0.0002174\text{m} = \mathbf{0.2174mm}$$

Then

$$T_{\max} = T(0.0002174) = \frac{T_2 - T_1}{L}y + T_1 + \frac{\mu V^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$= \frac{(40-15)\text{°C}}{0.0004\,\text{m}}(0.0002174\text{m}) + 15\text{°C} + \frac{(0.58\,\text{N}\cdot\text{s/m}^2)(12\,\text{m/s})^2}{2(0.145\text{W/m}\cdot\text{°C})}\left(\frac{0.0002174\text{m}}{0.0004\,\text{m}} - \frac{(0.0002174\text{m})^2}{(0.0004\,\text{m})^2}\right)$$

$$= \mathbf{100.0\,°C}$$

(*c*) Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-0) = -k\frac{T_2 - T_1}{L} - \frac{\mu V^2}{2L}$$

$$= -(0.145\,\text{W/m.°C})\frac{(40-15)\text{°C}}{0.0004\,\text{m}} - \frac{(0.58\,\text{N}\cdot\text{s/m}^2)(12\,\text{m/s})^2}{2(0.0004\text{m})}\left(\frac{1\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-1.135\times10^5\ W/m^2}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-2) = -k\frac{T_2 - T_1}{L} + \frac{\mu V^2}{2L}$$

$$= -(0.145\,\text{W/m.°C})\frac{(40-15)\text{°C}}{0.0004\,\text{m}} + \frac{(0.58\,\text{N}\cdot\text{s/m}^2)(12\,\text{m/s})^2}{2(0.0004\text{m})}\left(\frac{1\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{9.53\times10^4\ W/m^2}$$

Discussion A temperature rise of about 72.5°C confirms our suspicion that viscous dissipation is very significant. Calculations are done using oil properties at 27.5°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of about 64°C to improve accuracy.