6-39 The oil in a journal bearing is considered. The velocity and temperature distributions, the maximum temperature, the rate of heat transfer, and the mechanical power wasted in oil are to be determined.

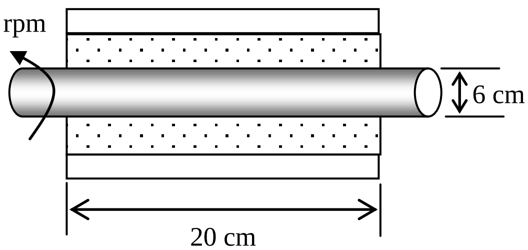
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil at 50°C are given to be

$$k = 0.17 \text{ W/m-K} \quad \text{and} \quad \mu = 0.05 \text{ N-s/m}^2$$

Analysis (*a*) Oil flow in journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \longrightarrow \dfrac{\partial u}{\partial x} = 0 \longrightarrow u = u(y)$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $\rho\left(u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y}\right) = \mu\dfrac{\partial^2 u}{\partial y^2} - \dfrac{\partial P}{\partial x} \longrightarrow \dfrac{d^2 u}{dy^2} = 0$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y) = C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Taking $x = 0$ at the surface of the bearing, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y) = \frac{y}{L}\mathcal{V}$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with viscous dissipation reduce to

Energy: $0 = k\dfrac{\partial^2 T}{\partial y^2} + \mu\left(\dfrac{\partial u}{\partial y}\right)^2 \longrightarrow k\dfrac{d^2 T}{dy^2} = -\mu\left(\dfrac{\mathcal{V}}{L}\right)^2$

since $\partial u/\partial y = \mathcal{V}/L$. Dividing both sides by *k* and integrating twice give

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2 + C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_0$ and $T(L) = T_0$ gives the temperature distribution to be

$$T(y) = T_0 + \frac{\mu\mathcal{V}^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for *y*,

$$\frac{dT}{dy} = \frac{\mu\mathcal{V}^2}{2kL}\left(1 - 2\frac{y}{L}\right) = 0 \longrightarrow y = \frac{L}{2}$$

Therefore, maximum temperature will occur at mid plane in the oil. The velocity and the surface area are

$$\mathrm{V} = \pi D\dot{n} = \pi(0.06\,\mathrm{m})(3000\,\mathrm{rev/min})\left(\frac{1\,\mathrm{min}}{60\,\mathrm{s}}\right) = 9.425\,\mathrm{m/s}$$

$$A = \pi D L_{bearing} = \pi(0.06\,\mathrm{m})(0.20\,\mathrm{m}) = 0.0377\,\mathrm{m}^2$$

The maximum temperature is

$$T_{\max} = T(L/2) = T_0 + \frac{\mu \mathrm{V}^2}{2k}\left(\frac{L/2}{L} - \frac{(L/2)^2}{L^2}\right)$$

$$= T_0 + \frac{\mu \mathrm{V}^2}{8k} = 50°\mathrm{C} + \frac{(0.05\,\mathrm{N\cdot s/m^2})(9.425\,\mathrm{m/s})^2}{8(0.17\,\mathrm{W/m\cdot °C})}\left(\frac{1\,\mathrm{W}}{1\,\mathrm{N\cdot m/s}}\right) = \mathbf{53.3°C}$$

(*b*) The rates of heat transfer are

$$\dot{Q}_0 = -kA\left.\frac{dT}{dy}\right|_{y=0} = -kA\frac{\mu \mathrm{V}^2}{2kL}(1-0) = -A\frac{\mu \mathrm{V}^2}{2L}$$

$$= -(0.0377\,\mathrm{m}^2)\frac{(0.05\,\mathrm{N\cdot s/m^2})(9.425\,\mathrm{m/s})^2}{2(0.0002\mathrm{m})}\left(\frac{1\,\mathrm{W}}{1\,\mathrm{N\cdot m/s}}\right) = \mathbf{-419\ W}$$

$$\dot{Q}_L = -kA\left.\frac{dT}{dy}\right|_{y=L} = -kA\frac{\mu \mathrm{V}^2}{2kL}(1-2) = A\frac{\mu \mathrm{V}^2}{2L} = -\dot{Q}_0 = \mathbf{419\ W}$$

Therefore, rates of heat transfer at the two plates are equal in magnitude but opposite in sign. The mechanical power wasted is equal to the rate of heat transfer.

$$\dot{W}_{\mathrm{mech}} = \dot{Q} = 2\times 419 = \mathbf{838\ W}$$

6-40 The oil in a journal bearing is considered. The velocity and temperature distributions, the maximum temperature, the rate of heat transfer, and the mechanical power wasted in oil are to be determined.

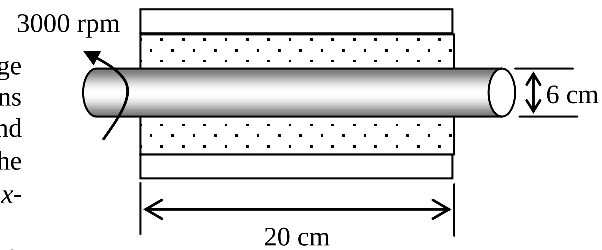
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil at 50°C are given to be

$k = 0.17$ W/m-K and $\mu = 0.05$ N-s/m^2

Analysis (*a*) Oil flow in journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \longrightarrow \dfrac{\partial u}{\partial x} = 0 \longrightarrow u = u(y)$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P / \partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2} = 0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y) = C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Taking $x = 0$ at the surface of the bearing, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y) = \frac{y}{L}\mathcal{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation reduce to

Energy: $$0 = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2 T}{dy^2} = -\mu\left(\frac{\mathcal{V}}{L}\right)^2$$

since $\partial u / \partial y = \mathcal{V} / L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy} = -\frac{\mu}{k}\left(\frac{\mathcal{V}}{L}\right)^2 y + C_3$$

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2 + C_3 y + C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $\quad T(0) = T_1 \longrightarrow C_4 = T_1$

B.C. 2: $y=L$ $\quad -k\left.\dfrac{dT}{dy}\right|_{y=L} = 0 \longrightarrow C_3 = \dfrac{\mu \mathcal{V}^2}{kL}$

Substituting the constants give the temperature distribution to be

$$T(y) = T_1 + \frac{\mu V^2}{kL}\left(y - \frac{y^2}{2L}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{\mu V^2}{kL}\left(1 - \frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = \frac{\mu V^2}{kL}\left(1 - \frac{y}{L}\right) = 0 \longrightarrow y = L$$

This result is also known from the second boundary condition. Therefore, maximum temperature will occur at the shaft surface, for $y = L$. The velocity and the surface area are

$$V = \pi D\dot{N} = \pi(0.06\,\text{m})(3000\,\text{rev/min})\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 9.425\,\text{m/s}$$

$$A = \pi DL_{bearing} = \pi(0.06\,\text{m})(0.20\,\text{m}) = 0.0377\,\text{m}^2$$

The maximum temperature is

$$T_{\max} = T(L) = T_1 + \frac{\mu V^2}{kL}\left(L - \frac{L^2}{2L}\right) = T_1 + \frac{\mu V^2}{k}\left(1 - \frac{1}{2}\right) = T_1 + \frac{\mu V^2}{2k}$$

$$= 50°\text{C} + \frac{(0.05\,\text{N}\cdot\text{s/m}^2)(9.425\,\text{m/s})^2}{2(0.17\,\text{W/m}\cdot°\text{C})}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{63.1°C}$$

(b) The rate of heat transfer to the bearing is

$$\dot{Q}_0 = -kA\left.\frac{dT}{dy}\right|_{y=0} = -kA\frac{\mu V^2}{kL}(1-0) = -A\frac{\mu V^2}{L}$$

$$= -(0.0377\,\text{m}^2)\frac{(0.05\,\text{N}\cdot\text{s/m}^2)(9.425\,\text{m/s})^2}{0.0002\text{m}}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-837\ W}$$

The rate of heat transfer to the shaft is zero. The mechanical power wasted is equal to the rate of heat transfer,

$$\dot{W}_{\text{mech}} = \dot{Q} = \mathbf{837\ W}$$

6-41

```
"!PROBLEM 6-41"

"GIVEN"
D=0.06 "[m]"
"N_dot=3000 rpm, parameter to be varied"
L_bearing=0.20 "[m]"
L=0.0002 "[m]"
T_0=50 "[C]"

"PROPERTIES"
k=0.17 "[W/m-K]"
mu=0.05 "[N-s/m^2]"

"ANALYSIS"
Vel=pi*D*N_dot*Convert(1/min, 1/s)
A=pi*D*L_bearing
T_max=T_0+(mu*Vel^2)/(8*k)
Q_dot=A*(mu*Vel^2)/(2*L)
W_dot_mech=Q_dot
```

N [rpm]	W_{mech} [W]
0	0
250	2.907
500	11.63
750	26.16
1000	46.51
1250	72.67
1500	104.7
1750	142.4
2000	186
2250	235.5
2500	290.7
2750	351.7
3000	418.6
3250	491.3
3500	569.8
3750	654.1
4000	744.2
4250	840.1
4500	941.9
4750	1049
5000	1163

6-42 A shaft rotating in a bearing is considered. The power required to rotate the shaft is to be determined for different fluids in the gap.

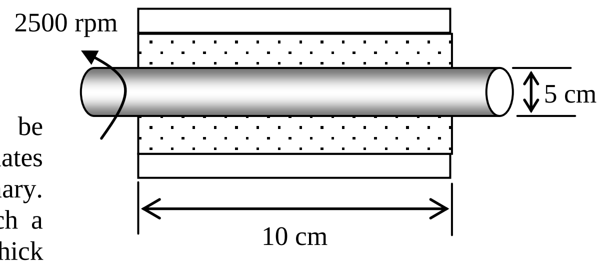
Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of air, water, and oil at 40°C are (Tables A-15, A-9, A-13)

Air: $\mu = 1.918\times10^{-5}$ N-s/m^2

Water: $\mu = 0.653\times10^{-3}$ N-s/m^2

Oil: $\mu = 0.212$ N-s/m^2



Analysis A shaft rotating in a bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. Therefore, we solve this problem considering such a flow with the plates separated by a L=0.5 mm thick fluid film similar to the problem given in Example 6-1. By simplifying and solving the continuity, momentum, and energy equations it is found in Example 6-1 that

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -\dot{Q}_L = -kA\left.\frac{dT}{dy}\right|_{y=0} = -kA\frac{\mu V^2}{2kL}(1-0) = -A\frac{\mu V^2}{2L} = -A\frac{\mu V^2}{2L}$$

First, the velocity and the surface area are

$$V = \pi D\dot{N} = \pi(0.05\,\text{m})(2500\,\text{rev/min})\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 6.545\,\text{m/s}$$

$$A = \pi DL_{\text{bearing}} = \pi(0.05\,\text{m})(0.10\,\text{m}) = 0.01571\,\text{m}^2$$

(a) Air:

$$\dot{W}_{\text{mech}} = -A\frac{\mu V^2}{2L} = -(0.01571\,\text{m}^2)\frac{(1.918\times10^{-5}\ \text{N}\cdot\text{s/m}^2)(6.545\,\text{m/s})^2}{2(0.0005\text{m})}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-0.013\ W}$$

(b) Water:

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -A\frac{\mu V^2}{2L} = -(0.01571\,\text{m}^2)\frac{(0.653\times10^{-3}\ \text{N}\cdot\text{s/m}^2)(6.545\,\text{m/s})^2}{2(0.0005\text{m})}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-0.44\ W}$$

(c) Oil:

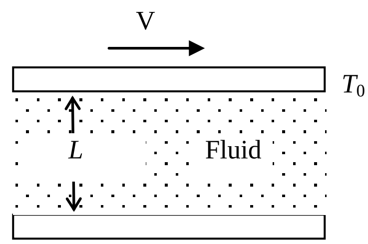
$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -A\frac{\mu V^2}{2L} = -(0.01571\,\text{m}^2)\frac{(0.212\,\text{N}\cdot\text{s/m}^2)(6.545\,\text{m/s})^2}{2(0.0005\text{m})}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-142.7\ W}$$

6-43 The flow of fluid between two large parallel plates is considered. The relations for the maximum temperature of fluid, the location where it occurs, and heat flux at the upper plate are to be obtained.

Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.

Analysis We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2}=0$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}\mathrm{V}$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

Energy: $0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2 T}{dy^2}=-\mu\left(\frac{\mathrm{V}}{L}\right)^2$

since $\partial u/\partial y = \mathrm{V}/L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{\mathrm{V}}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}\mathrm{V}\right)^2+C_3 y+C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $\quad -k\left.\frac{dT}{dy}\right|_{y=0}=0 \longrightarrow C_3=0$

B.C. 2: $y=L$ $\quad T(L)=T_0 \longrightarrow C_4=T_0+\frac{\mu \mathrm{V}^2}{2k}$

Substituting the constants give the temperature distribution to be

$$T(y)=T_0+\frac{\mu \mathrm{V}^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2}\, y$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2}\, y = 0 \longrightarrow y = 0$$

Therefore, maximum temperature will occur at the lower plate surface, and it s value is

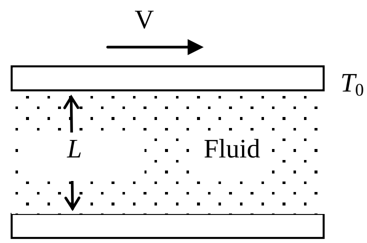
$$T_{\max} = T(0) = T_0 + \frac{\mu V^2}{2k}$$

.The heat flux at the upper plate is

$$\dot{q}_L = -k\frac{dT}{dy}\bigg|_{y=L} = k\,\frac{\mu V^2}{kL^2}\, L = \frac{\mu V^2}{L}$$

6-44 The flow of fluid between two large parallel plates is considered. Using the results of Problem 6-43, a relation for the volumetric heat generation rate is to be obtained using the conduction problem, and the result is to be verified.

Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.



Analysis The energy equation in Prob. 6-44 was determined to be

$$k\frac{d^2T}{dy^2} = -\mu\left(\frac{V}{L}\right)^2 \qquad (1)$$

The steady one-dimensional heat conduction equation with constant heat generation is

$$\frac{d^2T}{dy^2} + \frac{\dot{g}_0}{k} = 0 \qquad (2)$$

Comparing the two equation above, the volumetric heat generation rate is determined to be

$$\dot{g}_0 = \mu\left(\frac{V}{L}\right)^2$$

Integrating Eq. (2) twice gives

$$\frac{dT}{dy} = -\frac{\dot{g}_0}{k}y + C_3$$

$$T(y) = -\frac{\dot{g}_0}{2k}y^2 + C_3 y + C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $\qquad -k\left.\frac{dT}{dy}\right|_{y=0} = 0 \longrightarrow C_3 = 0$

B.C. 2: $y=L$ $\qquad T(L) = T_0 \longrightarrow C_4 = T_0 + \frac{\dot{g}_0}{2k}L^2$

Substituting, the temperature distribution becomes

$$T(y) = T_0 + \frac{\dot{g}_0 L^2}{2k}\left(1 - \frac{y^2}{L^2}\right)$$

Maximum temperature occurs at $y = 0$, and it value is

$$T_{\max} = T(0) = T_0 + \frac{\dot{g}_0 L^2}{2k}$$

which is equivalent to the result $T_{\max} = T(0) = T_0 + \frac{\mu V^2}{2k}$ obtained in Prob. 6-43.

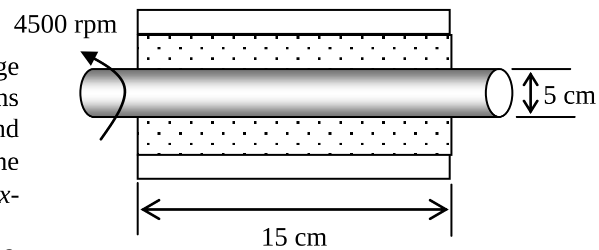
6-45 The oil in a journal bearing is considered. The bearing is cooled externally by a liquid. The surface temperature of the shaft, the rate of heat transfer to the coolant, and the mechanical power wasted are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil are given to be k = 0.14 W/m-K and μ = 0.03 N-s/m^2. The thermal conductivity of bearing is given to be k = 70 W/m-K.

Analysis (*a*) Oil flow in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to

Continuity: $\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0 \longrightarrow \dfrac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $\rho\left(u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}\right)=\mu\dfrac{\partial^2 u}{\partial y^2}-\dfrac{\partial P}{\partial x} \longrightarrow \dfrac{d^2u}{dy^2}=0$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}\mathcal{V}$$

where

$$\mathcal{V}=\pi D\dot{n}=\pi(0.05\,\text{m})(4500\,\text{rev/min})\left(\frac{1\,\text{min}}{60\,\text{s}}\right)=11.78\,\text{m/s}$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with viscous dissipation reduces to

Energy: $0=k\dfrac{\partial^2 T}{\partial y^2}+\mu\left(\dfrac{\partial u}{\partial y}\right)^2 \longrightarrow k\dfrac{d^2T}{dy^2}=-\mu\left(\dfrac{\mathcal{V}}{L}\right)^2$

since $\partial u/\partial y=\mathcal{V}/L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{\mathcal{V}}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2+C_3y+C_4$$

Applying the two boundary conditions give

B.C. 1: y=0 $\quad -k\left.\dfrac{dT}{dy}\right|_{y=0}=0 \longrightarrow C_3=0$

B.C. 2: $y=L$ $\quad T(L)=T_0 \longrightarrow C_4 = T_0 + \dfrac{\mu V^2}{2k}$

Substituting the constants give the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy}=\frac{-\mu V^2}{kL^2}y$$

.The heat flux at the upper surface is

$$\dot{q}_L=-k\left.\frac{dT}{dy}\right|_{y=L}=k\,\frac{\mu V^2}{kL^2}L=\frac{\mu V^2}{L}$$

Noting that heat transfer along the shaft is negligible, all the heat generated in the oil is transferred to the shaft, and the rate of heat transfer is

$$\dot{Q}=A_s\dot{q}_L=(\pi DW)\frac{\mu V^2}{L}=\pi(0.05\,\text{m})(0.15\text{m})\frac{(0.03\,\text{N}\cdot\text{s/m}^2)(11.78\,\text{m/s})^2}{0.0006\,\text{m}}=\mathbf{163.5\ W}$$

(b) This is equivalent to the rate of heat transfer through the cylindrical sleeve by conduction, which is expressed as

$$\dot{Q}=k\,\frac{2\pi W(T_0-T_s)}{\ln(D_0/D)}\quad\rightarrow\ (70\,\text{W/m}\cdot{}^\circ\text{C})\,\frac{2\pi(0.15\text{m})(T_0-40^\circ\text{C})}{\ln(8/5)}=163.5\,\text{W}$$

which gives the surface temperature of the shaft to be

T_o = **41.2°C**

(c) The mechanical power wasted by the viscous dissipation in oil is equivalent to the rate of heat generation,

$$\dot{W}_{lost}=\dot{Q}=\mathbf{163.5\ W}$$

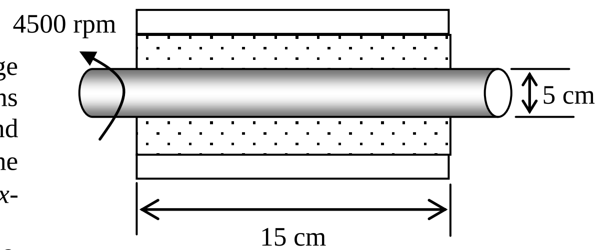
6-46 The oil in a journal bearing is considered. The bearing is cooled externally by a liquid. The surface temperature of the shaft, the rate of heat transfer to the coolant, and the mechanical power wasted are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil are given to be k = 0.14 W/m-K and μ = 0.03 N-s/m^2. The thermal conductivity of bearing is given to be k = 70 W/m-K.

Analysis (*a*) Oil flow in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to

Continuity: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2}=0$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = \mathcal{V}$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}\mathcal{V}$$

where

$$\mathcal{V}=\pi D\dot{n}=\pi(0.05\,\text{m})(4500\,\text{rev/min})\left(\frac{1\,\text{min}}{60\,\text{s}}\right)=11.78\,\text{m/s}$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with viscous dissipation reduces to

Energy: $0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2 T}{dy^2}=-\mu\left(\frac{\mathcal{V}}{L}\right)^2$

since $\partial u/\partial y=\mathcal{V}/L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{\mathcal{V}}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}\mathcal{V}\right)^2+C_3 y+C_4$$

Applying the two boundary conditions give

B.C. 1: y=0 $\quad -k\left.\frac{dT}{dy}\right|_{y=0}=0 \longrightarrow C_3=0$

B.C. 2: $y=L$ $\quad T(L)=T_0 \longrightarrow C_4 = T_0 + \dfrac{\mu V^2}{2k}$

Substituting the constants give the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy}=\frac{-\mu V^2}{kL^2}y$$

.The heat flux at the upper surface is

$$\dot{q}_L=-k\left.\frac{dT}{dy}\right|_{y=L}=k\frac{\mu V^2}{kL^2}L=\frac{\mu V^2}{L}$$

Noting that heat transfer along the shaft is negligible, all the heat generated in the oil is transferred to the shaft, and the rate of heat transfer is

$$\dot{Q}=A_s\dot{q}_L=(\pi DW)\frac{\mu V^2}{L}=\pi(0.05\,\text{m})(0.15\text{m})\frac{(0.03\text{N}\cdot\text{s/m}^2)(11.78\,\text{m/s})^2}{0.001\text{m}}=\mathbf{98.1\,W}$$

(b) This is equivalent to the rate of heat transfer through the cylindrical sleeve by conduction, which is expressed as

$$\dot{Q}=k\frac{2\pi W(T_0-T_s)}{\ln(D_0/D)}\quad\rightarrow(70\,\text{W/m}\cdot{}^\circ\text{C})\frac{2\pi(0.15\text{m})(T_0\text{ - }40^\circ\text{C})}{\ln(8/5)}=98.1\,\text{W}$$

which gives the surface temperature of the shaft to be

T_o = **40.7°C**

(c) The mechanical power wasted by the viscous dissipation in oil is equivalent to the rate of heat generation,

$$\dot{W}_{lost}=\dot{Q}=\mathbf{98.1\,W}$$

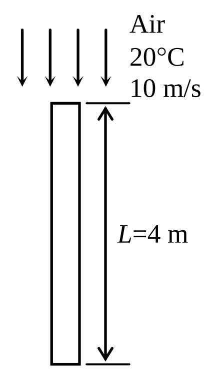
Momentum and Heat Transfer Analogies

6-47C Reynolds analogy is expressed as $C_{f,x}\dfrac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x$. It allows us to calculate the heat transfer coefficient from a knowledge of friction coefficient. It is limited to flow of fluids with a Prandtl number of near unity (such as gases), and negligible pressure gradient in the flow direction (such as flow over a flat plate).

6-48C Modified Reynolds analogy is expressed as $C_{f,x}\dfrac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x\,\mathrm{Pr}^{-1/3}$ or $\dfrac{C_{f,x}}{2} = \dfrac{h_x}{\rho C_p \mathcal{V}}\mathrm{Pr}^{2/3} \equiv j_H$. It allows us to calculate the heat transfer coefficient from a knowledge of friction coefficient. It is valid for a Prandtl number range of 0.6 < Pr < 60. This relation is developed using relations for laminar flow over a flat plate, but it is also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients.

6-49 A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection heat transfer coefficient and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at 20°C and 1 atm are (Table A-15)

$$\rho = 1.204 \text{ kg/m}^3, \quad C_p = 1.007 \text{ kJ/kg-K}, \quad \mathrm{Pr} = 0.7309$$

Analysis The flow is along the 4-m side of the plate, and thus the characteristic length is L = 4 m. Both sides of the plate is exposed to air flow, and thus the total surface area is

$$A_s = 2WL = 2(4\text{ m})(4\text{ m}) = 32\text{ m}^2$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} \longrightarrow C_f = \frac{F_f}{\rho A_s V^2/2} = \frac{2.4\text{ N}}{(1.204\text{kg/m}^3)(32\text{ m}^2)(10\text{ m/s})^2/2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 0.006229$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy to be

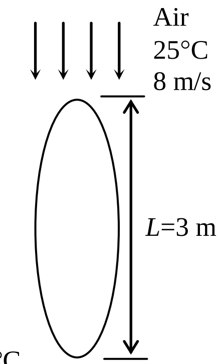
$$h = \frac{C_f}{2}\frac{\rho V C_p}{\mathrm{Pr}^{2/3}} = \frac{0.006229}{2}\frac{(1.204\text{ kg/m}^3)(10\text{ m/s})(1007\text{ J/kg}\cdot{}^\circ\text{C})}{(0.7309)^{2/3}} = \mathbf{46.54\ W/m^2\cdot C}$$

Them the rate of heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (46.54\text{ W/m}^2\cdot{}^\circ\text{C})(32\text{ m}^2)(80-20)^\circ\text{C} = \mathbf{89{,}356\ W}$$

6-50 A metallic airfoil is subjected to air flow. The average friction coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at 25°C and 1 atm are (Table A-15)

$$\rho = 1.184 \text{ kg/m}^3, \quad C_p = 1.007 \text{ kJ/kg-K}, \quad \text{Pr} = 0.7296$$

Analysis First, we determine the rate of heat transfer from

$$\dot{Q} = \frac{mC_{\text{p,airfoil}}(T_2 - T_1)}{\Delta t} = \frac{(50\ \text{kg})(500\,\text{J/kg}\cdot{}^\circ\text{C})(160-150)^\circ\text{C}}{(2\times 60\,\text{s})} = 2083\,\text{W}$$

Then the average heat transfer coefficient is

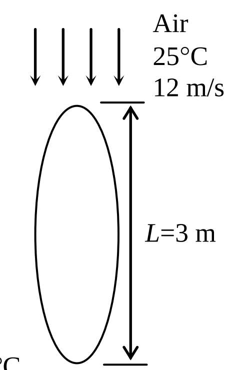
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{2083\,\text{W}}{(12\,\text{m}^2)(155-25)^\circ\text{C}} = 1.335\,\text{W/m}^2\cdot{}^\circ\text{C}$$

where the surface temperature of airfoil is taken as its average temperature, which is (150+160)/2=155°C. The average friction coefficient of the airfoil is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho \text{V} C_p} = \frac{2(1.335\,\text{W/m}^2\cdot{}^\circ\text{C})(0.7296)^{2/3}}{(1.184\,\text{kg/m}^3)(8\,\text{m/s})(1007\,\text{J/kg}\cdot{}^\circ\text{C})} = \mathbf{0.000227}$$

6-51 A metallic airfoil is subjected to air flow. The average friction coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at 25°C and 1 atm are (Table A-15)

$$\rho = 1.184 \text{ kg/m}^3, \quad C_p = 1.007 \text{ kJ/kg-K}, \quad \text{Pr} = 0.7296$$

Analysis First, we determine the rate of heat transfer from

$$\dot{Q} = \frac{mC_{\text{p,airfoil}}(T_2 - T_1)}{\Delta t} = \frac{(50\ \text{kg})(500\,\text{J/kg}\cdot{}^\circ\text{C})(160-150)^\circ\text{C}}{(2\times 60\,\text{s})} = 2083\,\text{W}$$

Then the average heat transfer coefficient is

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{2083\,\text{W}}{(12\,\text{m}^2)(155-25)^\circ\text{C}} = 1.335\,\text{W/m}^2\cdot{}^\circ\text{C}$$

where the surface temperature of airfoil is taken as its average temperature, which is (150+160)/2=155°C. The average friction coefficient of the airfoil is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho \text{V} C_p} = \frac{2(1.335\,\text{W/m}^2\cdot{}^\circ\text{C})(0.7296)^{2/3}}{(1.184\,\text{kg/m}^3)(12\,\text{m/s})(1007\,\text{J/kg}\cdot{}^\circ\text{C})} = \mathbf{0.0001512}$$

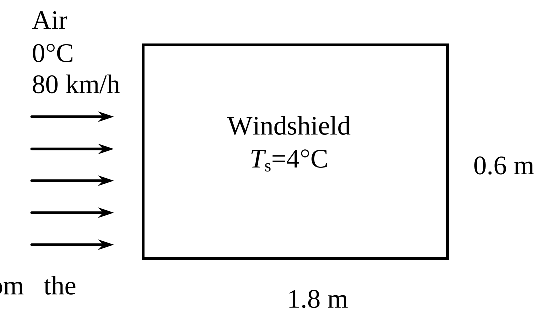
6-52 The windshield of a car is subjected to parallel winds. The drag force the wind exerts on the windshield is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at 0°C and 1 atm are (Table A-15)

ρ = 1.292 kg/m^3, C_p =1.006 kJ/kg-K, Pr = 0.7362

Analysis The average heat transfer coefficient is



$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{50\text{ W}}{(0.6\times 1.8\text{ m}^2)(4-0)°\text{C}} = 11.57\text{ W/m}^2\cdot°\text{C}$$

The average friction coefficient is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho \text{V} C_p} = \frac{2(11.57\text{ W/m}^2\cdot°\text{C})(0.7362)^{2/3}}{(1.292\text{ kg/m}^3)(80/3.6\text{ m/s})(1006\text{J/kg}\cdot°\text{C})} = 0.0006534$$

The drag force is determined from

$$F_f = C_f A_s \frac{\rho \text{V}^2}{2} = 0.0006534(0.6\times 1.8\text{ m}^2)\frac{(1.292\text{kg/m}^3)(80/3.6\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg.m/s}^2}\right) = \mathbf{0.225N}$$

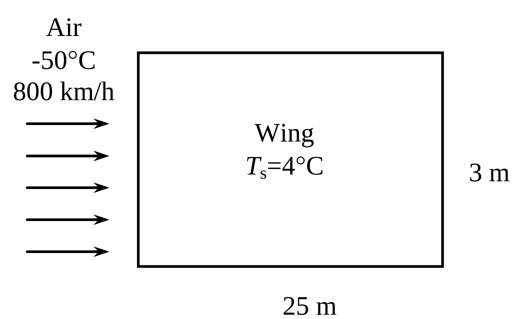
6-53 An airplane cruising is considered. The average heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at -50°C and 1 atm are (Table A-15)

C_p =0.999 kJ/kg-K Pr = 0.7440

The density of air at -50°C and 26.5 kPa is



$$\rho = \frac{P}{RT} = \frac{26.5\text{ kPa}}{(0.287\text{ kJ/kg.K})(-50+273)\text{K}} = 0.4141\text{kg/m}^3$$

Analysis The average heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h = \frac{C_f}{2}\frac{\rho \text{V} C_p}{\text{Pr}^{2/3}} = \frac{0.0016}{2}\frac{(0.4141\text{kg/m}^3)(800/3.6\text{ m/s})(999\text{J/kg}\cdot°\text{C})}{(0.7440)^{2/3}} = \mathbf{89.6\ W/m^2\cdot C}$$

6-54, 6-55 Design and Essay Problems
