Special Topic: Thermal Insulation

7-73C Thermal insulation is a material that is used primarily to provide resistance to heat flow. It differs from other kinds of insulators in that the purpose of an electrical insulator is to halt the flow of electric current, and the purpose of a sound insulator is to slow down the propagation of sound waves.

7-74C In *cold surfaces* such as chilled water lines, refrigerated trucks, and air conditioning ducts, insulation saves energy since the source of "coldness" is *refrigeration* that requires energy input. In this case heat is transferred from the surroundings to the cold surfaces, and the refrigeration unit must now work harder and longer to make up for this heat gain and thus it must consume more electrical energy.

7-75C The *R*-value of insulation is the *thermal resistance* of the insulating material *per unit surface area*. For *flat insulation* the *R*-value is obtained by simply dividing the thickness of the insulation by its thermal conductivity. That is, *R*-value = L/k. Doubling the thickness L doubles the *R*-value of flat insulation.

7-76C The R-value of an insulation represents the thermal resistance of insulation *per unit surface area* (or per unit length in the case of pipe insulation).

7-77C Superinsulations are obtained by using layers of highly reflective sheets separated by glass fibers in an evacuated space. Radiation between two surfaces is inversely proportional to the number of sheets used and thus heat loss by radiation will be very low by using this highly reflective sheets. Evacuating the space between the layers forms a vacuum which minimize conduction or convection through the air space.

7-78C Yes, hair or any other cover reduces heat loss from the head, and thus serves as insulation for the head. The insulating ability of hair or feathers is most visible in birds and hairy animals.

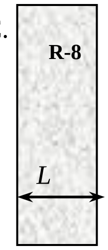
7-79C The primary reasons for insulating are energy conservation, personnel protection and comfort, maintaining process temperature, reducing temperature variation and fluctuations, condensation and corrosion prevention, fire protection, freezing protection, and reducing noise and vibration.

7-80C The *optimum* thickness of insulation is the thickness that corresponds to a minimum combined cost of insulation and heat lost. The cost of insulation increases roughly linearly with thickness while the cost of heat lost decreases exponentially. The total cost, which is the sum of the two, decreases first, reaches a minimum, and then increases. The thickness that corresponds to the minimum total cost is the optimum thickness of insulation, and this is the recommended thickness of insulation to be installed.

7-81 The thickness of flat *R*-8 insulation in SI units is to be determined when the thermal conductivity of the material is known.

Assumptions Thermal properties are constant.

Properties The thermal conductivity of the insulating material is given to be k = 0.04 W/m·°C.



Analysis The thickness of flat *R*-8 insulation (in m^2.°C/W) is determined from the definition of *R*-value to be

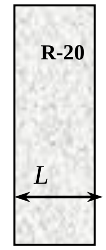
$$R_{\text{value}} = \frac{L}{k} \;\rightarrow\; L = R_{\text{value}}k = (8\ \text{m}^2.°\text{C / W})(0.04\ \text{W / m.°C}) = \mathbf{0.32\ m}$$

7-82E The thickness of flat *R*-20 insulation in English units is to be determined when the thermal conductivity of the material is known.

Assumptions Thermal properties are constant.

Properties The thermal conductivity of the insulating material is given to be k = 0.02 Btu/h·ft·°F.

Analysis The thickness of flat *R*-20 insulation (in h·ft^2·°F/Btu) is determined from the definition of *R*-value to be



$$R_{\text{value}} = \frac{L}{k} \rightarrow L = R_{\text{value}}k = (20\,\text{h.ft}^2.°\text{F/Btu})(0.02\ \text{Btu/h.ft.°F}) = \mathbf{0.4\,ft}$$

7-83 A steam pipe is to be covered with enough insulation to reduce the exposed surface temperature to 30°C . The thickness of insulation that needs to be installed is to be determined.

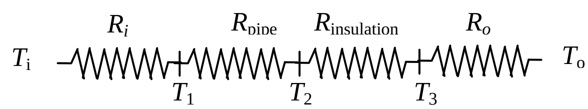
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 52 W/m·°C for cast iron pipe and k = 0.038 W/m·°C for fiberglass insulation.

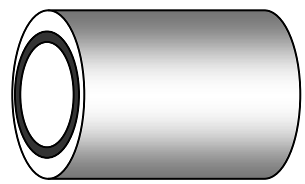
Analysis The thermal resistance network for this problem involves 4 resistances in series. The inner radius of the pipe is r_1 = 2.0 cm and the outer radius of the pipe and thus the inner radius of insulation is r_2 = 2.3 cm. Letting r_3 represent the outer radius of insulation, the areas of the surfaces exposed to convection for a L = 1 m long section of the pipe become

$$A_1 = 2\pi r_1 L = 2\pi(0.02 \text{ m})(1 \text{ m}) = 0.1257 \text{ m}^2$$

$$A_3 = 2\pi r_3 L = 2\pi r_3 (1 \text{ m}) = 2\pi r_3 \text{ m}^2 \quad (r_3 \text{ in m})$$



Then the individual thermal resistances are determined to be



$$R_i = R_{\text{conv},1} = \frac{1}{h_i A_1} = \frac{1}{(80 \text{ W/m}^2.°\text{C})(0.1257 \text{m}^2)} = 0.9944°\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(0.023/0.02)}{2\pi(52 \text{ W/m.°C})(1 \text{m})} = 0.00043°\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.023)}{2\pi(0.038 \text{ W/m.°C})(1 \text{ m})} = 4.188\ln(r_3/0.023) \text{ °C/W}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_o A_3} = \frac{1}{(22 \text{ W/m}^2.°\text{C})(2\pi r_3 \text{ m}^2)} = \frac{1}{138.2 r_3} \text{ °C/W}$$

Noting that all resistances are in series, the total resistance is

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.09944 + 0.00043 + 4.188\ln(r_3/0.023) + 1/(138.2 r_3) \text{ °C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_i - T_o}{R_{\text{total}}} = \frac{(110-22)°\text{C}}{[0.09944 + 0.00043 + 4.188\ln(r_3/0.023) + 1/(138.2 r_3)]°\text{C/W}}$$

Noting that the outer surface temperature of insulation is specified to be 30°C, the rate of heat loss can also be expressed as

$$\dot{Q} = \frac{T_3 - T_o}{R_o} = \frac{(30-22)°\text{C}}{1/(138.2 r_3)°\text{C/W}} = 1106 r_3$$

Setting the two relations above equal to each other and solving for r_3 gives r_3 = 0.0362 m. Then the minimum thickness of fiberglass insulation required is

$t = r_3 - r_2$ = 0.0362 - 0.0230 = 0.0132 m = **1.32 cm**

Therefore, insulating the pipe with at least 1.32 cm thick fiberglass insulation will ensure that the outer surface temperature of the pipe will be at 30°C or below.

7-84 "!PROBLEM 7-84"

```
"GIVEN"
T_i=110 "[C]"
T_o=22 "[C]"
k_pipe=52 "[W/m-C]"
r_1=0.02 "[m]"
t_pipe=0.003 "[m]"
"T_s_max=30 [C], parameter to be varied"
h_i=80 "[W/m^2-C]"
h_o=22 "[W/m^2-C]"
k_ins=0.038 "[W/m-C]"

"ANALYSIS"
L=1 "[m], 1 m long section of the pipe is considered"
A_i=2*pi*r_1*L
A_o=2*pi*r_3*L
r_3=r_2+t_ins*Convert(cm, m) "t_ins is in cm"
r_2=r_1+t_pipe
R_conv_i=1/(h_i*A_i)
R_pipe=ln(r_2/r_1)/(2*pi*k_pipe*L)
R_ins=ln(r_3/r_2)/(2*pi*k_ins*L)
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_pipe+R_ins+R_conv_o
Q_dot=(T_i-T_o)/R_total
Q_dot=(T_s_max-T_o)/R_conv_o
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$T_{s, max}$ [C]	t_{ins} [cm]
24	4.45
26	2.489
28	1.733
30	1.319
32	1.055
34	0.871
36	0.7342
38	0.6285
40	0.5441
42	0.4751
44	0.4176
46	0.3688
48	0.327

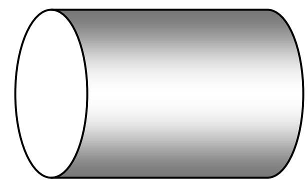
7-85 A cylindrical oven is to be insulated to reduce heat losses. The optimum thickness of insulation and the amount of money saved per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulation is one-dimensional. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** The surface temperature of the furnace and the heat transfer coefficient remain constant. **6** The surfaces of the cylindrical oven can be treated as plain surfaces since its diameter is greater than 1 m.

Properties The thermal conductivity of insulation is given to be $k = 0.038$ W/m·°C.

Analysis We treat the surfaces of this cylindrical furnace as plain surfaces since its diameter is greater than 1 m, and disregard the curvature effects. The exposed surface area of the furnace is

$$A_o = 2A_{\text{base}} + A_{\text{side}} = 2\pi r^2 + 2\pi rL = 2\pi(1.5\,\text{m})^2 + 2\pi(1.5\,\text{m})(6\,\text{m}) = 70.69\,\text{m}^2$$

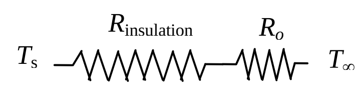


The rate of heat loss from the furnace before the insulation is installed is

$$\dot{Q} = h_o A_o (T_s - T_\infty) = (30\,\text{W/m}^2.°\text{C})(70.69\,\text{m}^2)(90-27)°\text{C} = 133{,}600\,\text{W}$$

Noting that the plant operates 52×80 = 4160 h/yr, the annual heat lost from the furnace is

$$Q = \dot{Q}\Delta t = (133.6\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 2.001\times 10^9\text{ kJ/yr}$$



The efficiency of the furnace is given to be 78 percent. Therefore, to generate this much heat, the furnace must consume energy (in the form of natural gas) at a rate of

$$Q_{\text{in}} = Q/\eta_{\text{oven}} = (2.001\times 10^9\text{ kJ/yr})/0.78 = 2.565\times 10^9\text{ kJ/yr} = 24{,}314\text{ therms/yr}$$

since 1 therm = 105,500 kJ. Then the annual fuel cost of this furnace before insulation becomes

$$\text{Annual Cost} = \text{Q}_{\text{in}} \times \text{Unit cost} = (24{,}314\text{ therm/yr})(\$0.50/\text{therm}) = \$12{,}157/\text{yr}$$

We expect the surface temperature of the furnace to increase, and the heat transfer coefficient to decrease somewhat when insulation is installed. We assume these two effects to counteract each other. Then the rate of heat loss for 1-cm thick insulation becomes

$$\dot{Q}_{\text{ins}} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{A_o(T_s - T_\infty)}{\dfrac{t_{\text{ins}}}{k_{\text{ins}}} + \dfrac{1}{h_o}} = \frac{(70.69\,\text{m}^2)(90-27)°\text{C}}{\dfrac{0.01\,\text{m}}{0.038\,\text{W/m.°C}} + \dfrac{1}{30\,\text{W/m}^2.°\text{C}}} = 15{,}021\,\text{W}$$

Also, the total amount of heat loss from the furnace per year and the amount and cost of energy consumption of the furnace become

$$Q_{\text{ins}} = \dot{Q}_{\text{ins}}\Delta t = (15.021\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 2.249\times 10^8\text{ kJ/yr}$$

$$Q_{\text{in,ins}} = Q_{\text{ins}}/\eta_{\text{oven}} = (2.249\times 10^8\text{ kJ/yr})/0.78 = 2.884\times 10^8\text{ kJ/yr} = 2734\text{ therms}$$

$$\text{Annual Cost} = \text{Q}_{\text{in,ins}} \times \text{Unit cost} = (2734\text{ therm/yr})(\$0.50/\text{therm}) = \$1367/\text{yr}$$

Cost savings = Energy cost w/o insulation - Energy cost w/insulation = 12,157 - 1367 = $10,790/yr

The unit cost of insulation is given to be $\$10/\text{m}^2$ per cm thickness, plus $\$30/\text{m}^2$ for labor. Then the total cost of insulation becomes

$$\text{Insulation Cost} = (\text{Unit cost})(\text{Surface area}) = [(\$10/\text{cm})(1\text{ cm}) + \$30/\text{m}^2](70.69\text{ m}^2) = \$2828$$

To determine the thickness of insulation whose cost is equal to annual energy savings, we repeat the calculations above for 2, 3, ... 15 cm thick insulations, and list the results in the table below.

Insulation thickness	Rate of heat loss W	Cost of heat lost $/yr	Cost savings $/yr	Insulation cost $
0 cm	133,600	12,157	0	0
1 cm	15,021	1367	10,790	2828
5 cm	3301	300	11,850	3535
10 cm	1671	152	12,005	9189
11 cm	1521	138	12,019	9897
12 cm	1396	127	12,030	10,604
13 cm	1289	117	12,040	11,310
14 cm	1198	109	12,048	12,017
15 cm	1119	102	12,055	12,724

Therefore, the thickest insulation that will pay for itself in one year is the one whose thickness is **14 cm**.

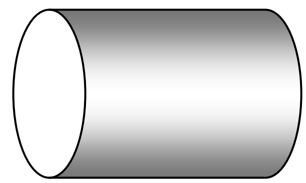
7-86 A cylindrical oven is to be insulated to reduce heat losses. The optimum thickness of insulation and the amount of money saved per year are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulation is one-dimensional. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** The surface temperature of the furnace and the heat transfer coefficient remain constant. **6** The surfaces of the cylindrical oven can be treated as plain surfaces since its diameter is greater than 1 m.

Properties The thermal conductivity of insulation is given to be k = 0.038 W/m·°C.

Analysis We treat the surfaces of this cylindrical furnace as plain surfaces since its diameter is greater than 1 m, and disregard the curvature effects. The exposed surface area of the furnace is

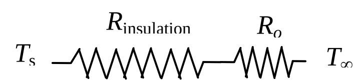
$$A_o = 2A_{\text{base}} + A_{\text{side}} = 2\pi r^2 + 2\pi rL = 2\pi(1.5\,\text{m})^2 + 2\pi(1.5\,\text{m})(6\,\text{m}) = 70.69\,\text{m}^2$$



The rate of heat loss from the furnace before the insulation is installed is

$$\dot{Q} = h_o A_o (T_s - T_\infty) = (30\,\text{W/m}^2.°\text{C})(70.69\,\text{m}^2)(75-27)°\text{C} = 101{,}794\,\text{W}$$

Noting that the plant operates 52×80 = 4160 h/yr, the annual heat lost from the furnace is



$$Q = \dot{Q}\Delta t = (101.794\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 1.524\times 10^9\text{ kJ/yr}$$

The efficiency of the furnace is given to be 78 percent. Therefore, to generate this much heat, the furnace must consume energy (in the form of natural gas) at a rate of

$$Q_{\text{in}} = Q/\eta_{\text{oven}} = (1.524\times 10^9\text{ kJ/yr})/0.78 = 1.954\times 10^9\text{ kJ/yr} = 18{,}526\text{ therms/yr}$$

since 1 therm = 105,500 kJ. Then the annual fuel cost of this furnace before insulation becomes

$$\text{Annual Cost} = \text{Q}_{\text{in}} \times \text{Unit cost} = (18{,}526\text{ therm/yr})(\$0.50/\text{therm}) = \$9{,}263/\text{yr}$$

We expect the surface temperature of the furnace to increase, and the heat transfer coefficient to decrease somewhat when insulation is installed. We assume these two effects to counteract each other. Then the rate of heat loss for 1-cm thick insulation becomes

$$\dot{Q}_{\text{ins}} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{A_o(T_s - T_\infty)}{\dfrac{t_{\text{ins}}}{k_{\text{ins}}} + \dfrac{1}{h_o}} = \frac{(70.69\,\text{m}^2)(75-27)°\text{C}}{\dfrac{0.01\,\text{m}}{0.038\,\text{W/m.°C}} + \dfrac{1}{30\,\text{W/m}^2.°\text{C}}} = 11{,}445\,\text{W}$$

Also, the total amount of heat loss from the furnace per year and the amount and cost of energy consumption of the furnace become

$$Q_{\text{ins}} = \dot{Q}_{\text{ins}}\Delta t = (11.445\text{ kJ/s})(4160\times 3600\text{ s/yr}) = 1.714\times 10^8\text{ kJ/yr}$$

$$Q_{\text{in,ins}} = Q_{\text{ins}}/\eta_{\text{oven}} = (1.714\times 10^8\text{ kJ/yr})/0.78 = 2.197\times 10^8\text{ kJ/yr} = 2082\text{ therms}$$

$$\text{Annual Cost} = Q_{\text{in,ins}} \times \text{Unit cost} = (2082\text{ therm/yr})(\$0.50/\text{therm}) = \$1041/\text{yr}$$

Cost savings = Energy cost w/o insulation - Energy cost w/insulation = 9263 - 1041 = \$8222/yr

The unit cost of insulation is given to be \$10/m^2 per cm thickness, plus \$30/m^2 for labor. Then the total cost of insulation becomes

$$\text{Insulation Cost} = (\text{Unit cost})(\text{Surface area}) = [(\$10/\text{cm})(1\text{ cm}) + \$30/\text{m}^2](70.69\text{ m}^2) = \$2828$$

To determine the thickness of insulation whose cost is equal to annual energy savings, we repeat the calculations above for 2, 3, ... 15 cm thick insulations, and list the results in the table below.

Insulation Thickness	Rate of heat loss W	Cost of heat lost $/yr	Cost savings $/yr	Insulation cost $
0 cm	101,794	9263	0	0
1 cm	11,445	1041	8222	2828
5 cm	2515	228	9035	3535
9 cm	1413	129	9134	8483
10 cm	1273	116	9147	9189
11 cm	1159	105	9158	9897
12 cm	1064	97	9166	10,604

Therefore, the thickest insulation that will pay for itself in one year is the one whose thickness is **9 cm**. The 10-cm thick insulation will come very close to paying for itself in one year.

7-87E Steam is flowing through an insulated steel pipe, and it is proposed to add another 1-in thick layer of fiberglass insulation on top of the existing one to reduce the heat losses further and to save energy and money. It is to be determined if the new insulation will pay for itself within 2 years.

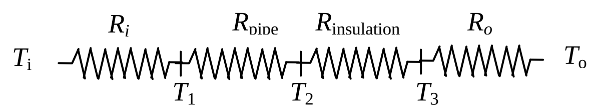
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The heat transfer coefficients remain constant. **5** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 8.7 Btu/h·ft·°F for steel pipe and k = 0.020 Btu/h·ft·°F for fiberglass insulation.

Analysis The inner radius of the pipe is r_1 = 1.75 in, the outer radius of the pipe is r_2 = 2 in, and the outer radii of the existing and proposed insulation layers are r_3 = 3 in and 4 in, respectively. Considering a unit pipe length of L = 1 ft, the individual thermal resistances are determined to be

$$R_i = R_{\text{conv},1} = \frac{1}{h_i A_1} = \frac{1}{h_i(2\pi r_1 L)} = \frac{1}{(30\text{ Btu/h.ft}^2.^\circ\text{F})[2\pi(1.75/12\text{ ft})(1\text{ ft})]} = 0.0364\text{ h.}^\circ\text{F/Btu}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2/1.75)}{2\pi(8.7\text{ Btu/h.ft.}^\circ\text{F})(1\text{ ft})} = 0.00244\text{ h.}^\circ\text{F/Btu}$$



Current Case:

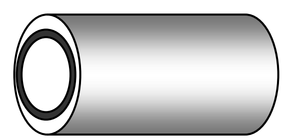
$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_{\text{ins}} L} = \frac{\ln(3/2)}{2\pi(0.020\text{ Btu/h.ft.}^\circ\text{F})(1\text{ ft})} = 3.227\text{ h.}^\circ\text{F/Btu}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_o A_3} = \frac{1}{h_o(2\pi r_3)} = \frac{1}{(5\text{ Btu/h.ft}^2.^\circ\text{F})[2\pi(3/12\text{ ft})(1\text{ ft})]} = 0.1273\text{ h.}^\circ\text{F/Btu}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q}_{\text{current}} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o} = \frac{(400-60)^\circ\text{F}}{(0.0364 + 0.00244 + 3.227 + 0.1273)\text{ h.}^\circ\text{F/Btu}} = 100.2\text{ Btu/h}$$

Proposed Case:



$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_{ins} L} = \frac{\ln(4/2)}{2\pi(0.020\text{ Btu/h.ft.}^\circ\text{F})(1\text{ ft})} = 5.516\text{ h.}^\circ\text{F/Btu}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_o A_3} = \frac{1}{h_o(2\pi r_3)} = \frac{1}{(5\text{ Btu/h.ft}^2.^\circ\text{F})[2\pi(4/12\text{ ft})(1\text{ ft})]} = 0.0955\text{ h.}^\circ\text{F/Btu}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q}_{\text{prop}} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_i - T_o}{R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o} = \frac{(400-60)^\circ\text{F}}{(0.0364 + 0.00244 + 5.516 + 0.0955)\text{ h.}^\circ\text{F/Btu}} = 60.2\text{ Btu/h}$$

Therefore, the amount of energy and money saved by the additional insulation per year are

$$\dot{Q}_{\text{saved}} = \dot{Q}_{\text{prop}} - \dot{Q}_{\text{current}} = 100.2 - 60.2 = 40.0\text{ Btu/h}$$

$$Q_{\text{saved}} = \dot{Q}_{\text{saved}}\Delta t = (40.0\text{ Btu/h})(8760\text{ h/yr}) = 350{,}400\text{ Btu/yr}$$

$$\text{Money saved} = Q_{\text{saved}} \times (\text{Unit cost}) = (350{,}400\text{ Btu/yr})(\$0.01/1000\text{ Btu}) = \$3.504/\text{yr}$$

or $7.01 per 2 years, which is barely more than the $7.0 minimum required. But the criteria is satisfied, and the proposed additional insulation is **justified**.

7-88 The plumbing system of a plant involves some section of a plastic pipe exposed to the ambient air. The pipe is to be insulated with adequate fiber glass insulation to prevent freezing of water in the pipe. The thickness of insulation that will protect the water from freezing under worst conditions is to be determined.

Assumptions **1** Heat transfer is transient, but can be treated as steady at average conditions. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The water in the pipe is stationary, and its initial temperature is 15°C. **5** The thermal contact resistance at the interface is negligible. **6** The convection resistance inside the pipe is negligible.

Properties The thermal conductivities are given to be k = 0.16 W/m·°C for plastic pipe and k = 0.035 W/m·°C for fiberglass insulation. The density and specific heat of water are to be ρ = 1000 kg/m^3 and C_p = 4.18 kJ/kg.°C (Table A-15).

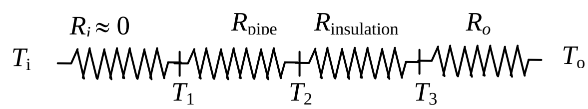
Analysis The inner radius of the pipe is r_1 = 3.0 cm and the outer radius of the pipe and thus the inner radius of insulation is r_2 = 3.3 cm. We let r_3 represent the outer radius of insulation. Considering a 1-m section of the pipe, the amount of heat that must be transferred from the water as it cools from 15 to 0°C is determined to be

$$m = \rho V = \rho(\pi r_1^{\,2} L) = (1000\,\text{kg/m}^3)[\pi(0.03\text{m})^2(1\text{m})] = 2.827\,\text{kg}$$

$$Q_{\text{total}} = mC_p\Delta T = (2.827\,\text{kg})(4.18\,\text{kJ/kg.°C})(15 - 0)\text{°C} = 177.3\,\text{kJ}$$

Then the average rate of heat transfer during 60 h becomes

$$\dot{Q}_{\text{ave}} = \frac{Q_{\text{total}}}{\Delta t} = \frac{177{,}300\ \text{J}}{(60\times 3600\ \text{s})} = 0.821\ \text{W}$$

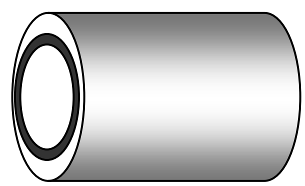


The individual thermal resistances are

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_{\text{pipe}} L} = \frac{\ln(0.033/0.03)}{2\pi(0.16\ \text{W/m.°C})(1\ \text{m})} = 0.0948\ \text{°C/W}$$

$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.033)}{2\pi(0.035\ \text{W/m.°C})(1\ \text{m})} = 4.55\ln(r_3/0.033)\ \text{°C/W}$$

$$R_o = R_{\text{conv}} = \frac{1}{h_o A_3} = \frac{1}{(30\ \text{W/m}^2\text{.°C})(2\pi r_3\ \text{m}^2)} = \frac{1}{188.5 r_3}\ \text{°C/W}$$



Then the rate of average heat transfer from the water can be expressed as

$$\dot{Q} = \frac{T_{i,ave} - T_o}{R_{\text{total}}} \;\rightarrow\; 0.821\ \text{W} = \frac{[7.5 - (-10)]\text{°C}}{[0.0948 + 4.55\ln(r_3/0.033) + 1/(188.5 r_3)]\text{°C/W}} \;\rightarrow\; r_3 = 3.50\ \text{m}$$

Therefore, the minimum thickness of fiberglass needed to protect the pipe from freezing is

$t = r_3 - r_2$ = 3.50 - 0.033 = **3.467 m**

which is too large. Installing such a thick insulation is not practical, however, and thus other freeze protection methods should be considered.

7-89 The plumbing system of a plant involves some section of a plastic pipe exposed to the ambient air. The pipe is to be insulated with adequate fiber glass insulation to prevent freezing of water in the pipe. The thickness of insulation that will protect the water from freezing more than 20% under worst conditions is to be determined.

Assumptions **1** Heat transfer is transient, but can be treated as steady at average conditions. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The water in the pipe is stationary, and its initial temperature is 15°C. **5** The thermal contact resistance at the interface is negligible. **6** The convection resistance inside the pipe is negligible.

Properties The thermal conductivities are given to be k = 0.16 W/m·°C for plastic pipe and k = 0.035 W/m·°C for fiberglass insulation. The density and specific heat of water are to be ρ = 1000 kg/m^3 and C_p = 4.18 kJ/kg.°C (Table A-15).

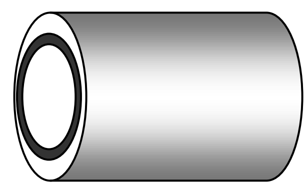
Analysis The inner radius of the pipe is r_1 = 3.0 cm and the outer radius of the pipe and thus the inner radius of insulation is r_2 = 3.3 cm. We let r_3 represent the outer radius of insulation. The latent heat of freezing of water is 33.7 kJ/kg. Considering a 1-m section of the pipe, the amount of heat that must be transferred from the water as it cools from 15 to 0°C is determined to be

$$m = \rho V = \rho(\pi r_1^{\,2} L) = (1000\,\text{kg/m}^3)[\pi(0.03\text{m})^2(1\,\text{m})] = 2.827\,\text{kg}$$

$$Q_{\text{total}} = mC_p\Delta T = (2.827\,\text{kg})(4.18\,\text{kJ/kg.°C})(15-0)\text{°C} = 177.3\,\text{kJ}$$

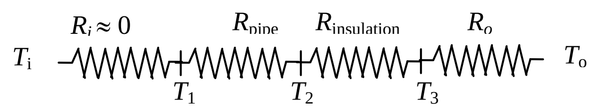
$$Q_{\text{freezing}} = 0.2\times m h_{if} = 0.2\times(2.827\,\text{kg})(333.7\text{ kJ/kg}) = 188.7\text{ kJ}$$

$$Q_{\text{total}} = Q_{\text{cooling}} + Q_{\text{freezing}} = 177.3 + 188.7 = 366.0\text{ kJ}$$



Then the average rate of heat transfer during 60 h becomes

$$\dot{Q}_{\text{ave}} = \frac{Q_{\text{total}}}{\Delta t} = \frac{366{,}000\text{ J}}{(60\times 3600\text{ s})} = 1.694\text{ W}$$



The individual thermal resistances are

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_{\text{pipe}} L} = \frac{\ln(0.033/0.03)}{2\pi(0.16\text{ W/m.°C})(1\text{ m})} = 0.0948\text{ °C/W}$$

$$R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.033)}{2\pi(0.035\text{ W/m.°C})(1\text{ m})} = 4.55\ln(r_3/0.033)\text{ °C/W}$$

$$R_o = R_{\text{conv}} = \frac{1}{h_o A_3} = \frac{1}{(30\text{ W/m}^2\text{.°C})(2\pi r_3\text{ m}^2)} = \frac{1}{188.5 r_3}\text{ °C/W}$$

Then the rate of average heat transfer from the water can be expressed as

$$\dot{Q} = \frac{T_{i,ave} - T_o}{R_{\text{total}}} \;\rightarrow\; 1.694\text{ W} = \frac{[7.5-(-10)]\text{°C}}{[0.0948 + 4.55\ln(r_3/0.033) + 1/(188.5 r_3)]\text{°C/W}} \;\rightarrow\; r_3 = 0.312\text{ m}$$

Therefore, the minimum thickness of fiberglass needed to protect the pipe from freezing is

$t = r_3 - r_2$ = 0.312 - 0.033 = **0.279 m**

which is too large. Installing such a thick insulation is not practical, however, and thus other freeze protection methods should be considered.

Review Problems

7-90 Wind is blowing parallel to the walls of a house. The rate of heat loss from the wall is to be determined.

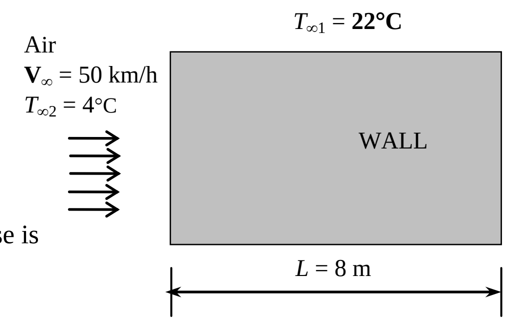
Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties Assuming a film temperature of T_f = 10°C for the outdoors, the properties of air are evaluated to be (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$

$$\upsilon = 1.426\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7336$$



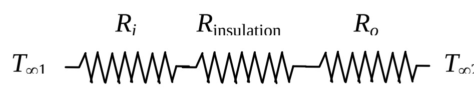
Analysis Air flows along 8-m side. The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(50\times1000/3600)\text{ m/s}](8\text{ m})}{1.426\times10^{-5}\text{ m}^2\text{/s}} = 7.792\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{h_0 L}{k} = (0.037\text{Re}_L{}^{0.8} - 871)\text{Pr}^{1/3} = [0.037(7.792\times10^6)^{0.8} - 871](0.7336)^{1/3} = 10{,}096$$

$$h_o = \frac{k}{L}Nu = \frac{0.02439\text{ W/m.°C}}{8\text{ m}}(10{,}096) = 30.78\text{ W/m}^2\text{.°C}$$



The thermal resistances are

$$A_s = wL = (3\text{ m})(8\text{ m}) = 24\text{ m}^2$$

$$R_i = \frac{1}{h_i A_s} = \frac{1}{(8\text{ W/m}^2\text{.°C})(24\text{ m}^2)} = 0.0052\text{°C/W}$$

$$R_{insulation} = \frac{(R-3.38)_{value}}{A_s} = \frac{3.38\text{ m}^2\text{.°C/W}}{24\text{ m}^2} = 0.1408\text{°C/W}$$

$$R_o = \frac{1}{h_o A_s} = \frac{1}{(30.78\text{ W/m}^2\text{.°C})(24\text{ m}^2)} = 0.0014\text{°C/W}$$

Then the total thermal resistance and the heat transfer rate through the wall are determined from

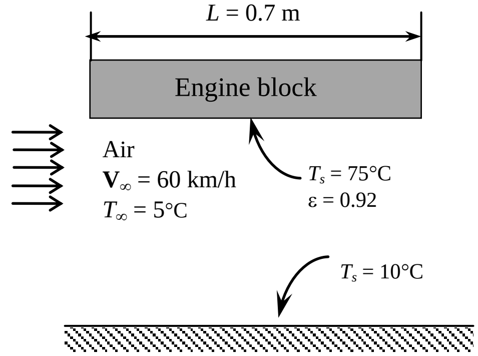
$$R_{total} = R_i + R_{insulation} + R_o = 0.0052 + 0.1408 + 0.0014 = 0.1474\text{°C/W}$$

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(22-4)\text{°C}}{0.1474\text{°C/W}} = \mathbf{122.1\text{ W}}$$

7-91 A car travels at a velocity of 60 km/h. The rate of heat transfer from the bottom surface of the hot automotive engine block is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm. **5** The flow is turbulent over the entire surface because of the constant agitation of the engine block. **6** The bottom surface of the engine is a flat surface.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (75+5)/2 = 40°C$ are (Table A-15)



$$k = 0.02662 \text{ W/m.°C}$$
$$\upsilon = 1.702\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7255$$

Analysis The Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(60\times1000/3600) \text{ m/s}](0.7 \text{ m})}{1.702\times10^{-5} \text{ m}^2/\text{s}} = 6.855\times10^5$$

which is less than the critical Reynolds number. But we will assume turbulent flow because of the constant agitation of the engine block.

$$Nu = \frac{hL}{k} = 0.037\text{Re}_L^{0.8}\text{Pr}^{1/3} = 0.037(6.855\times10^5)^{0.8}(0.7255)^{1/3} = 1551$$

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{0.7 \text{ m}}(1551) = 58.97 \text{ W/m}^2.°\text{C}$$

$$\dot{Q}_{conv} = hA_s(T_\infty - T_s) = (58.97 \text{ W/m}^2.°\text{C})[(0.6 \text{ m})(0.7 \text{ m})](75\text{-}5)°\text{C} = 1734 \text{ W}$$

The heat loss by radiation is then determined from Stefan-Boltzman law to be

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.92)(0.6 \text{ m})(0.7 \text{ m})(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(75+273 \text{K})^4 - (10+273 \text{K})^4\right] = 181 \text{W}$$

Then the total rate of heat loss from the bottom surface of the engine block becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 1734 + 181 = \mathbf{1915 \text{ W}}$$

The gunk will introduce an additional resistance to heat dissipation from the engine. The total heat transfer rate in this case can be calculated from

$$\dot{Q} = \frac{T_\infty - T_s}{\frac{1}{hA_s} + \frac{L}{kA_s}} = \frac{(75\text{-}5)°\text{C}}{\frac{1}{(58.97 \text{ W/m}^2.°\text{C})[(0.6 \text{ m})(0.7 \text{ m})]} + \frac{(0.002 \text{ m})}{(3 \text{ W/m.°C})(0.6 \text{ m}\times0.7 \text{ m})}} = 1668 \text{W}$$

The decrease in the heat transfer rate is

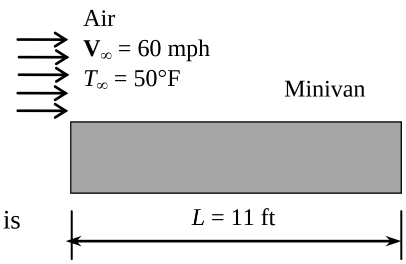
1734-1668 = **66 W**

7-92E A minivan is traveling at 60 mph. The rate of heat transfer to the van is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air flow is turbulent because of the intense vibrations involved. **5** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties Assuming a film temperature of T_f = 80°F, the properties of air are evaluated to be (Table A-15E)

$$k = 0.01481 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.1697\times10^{-3} \text{ ft}^2\text{/s}$$
$$\Pr = 0.7290$$



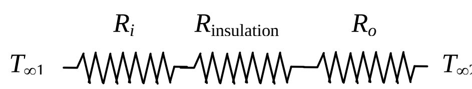
Analysis Air flows along 11 ft long side. The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(60\times5280/3600)\text{ ft/s}](11\text{ ft})}{0.1697\times10^{-3}\text{ ft}^2\text{/s}} = 5.704\times10^6$$

which is greater than the critical Reynolds number. The air flow is assumed to be entirely turbulent because of the intense vibrations involved. Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = \frac{h_o L}{k} = 0.037\text{Re}_L^{0.8}\Pr^{1/3} = 0.037(5.704\times10^6)^{0.8}(0.7290)^{1/3} = 8461$$

$$h_o = \frac{k}{L}Nu = \frac{0.01481\text{ Btu/h.ft.°F}}{11\text{ ft}}(8461) = 11.39\text{ Btu/h.ft}^2\text{.°F}$$



The thermal resistances are

$$A_s = 2[(3.2\text{ ft})(6\text{ ft}) + (3.2\text{ ft})(11\text{ ft}) + (6\text{ ft})(11\text{ ft})] = 240.8\text{ ft}^2$$

$$R_i = \frac{1}{h_i A_s} = \frac{1}{(1.2\text{ Btu/h.ft}^2\text{.°F})(240.8\text{ft}^2)} = 0.0035\text{ h.°F/Btu}$$

$$R_{insulation} = \frac{(R-3)_{value}}{A_s} = \frac{3\text{ h.ft}^2\text{.°F/Btu}}{(240.8\text{ft}^2)} = 0.0125\text{ h.°F/Btu}$$

$$R_o = \frac{1}{h_o A_s} = \frac{1}{(11.39\text{ Btu/h.ft}^2\text{.°F})(240.8\text{ft}^2)} = 0.0004\text{ h.°F/Btu}$$

Then the total thermal resistance and the heat transfer rate into the minivan are determined to be

$$R_{total} = R_i + R_{insulation} + R_o = 0.0035 + 0.0125 + 0.0004 = 0.0164\text{ h.°F/Btu}$$

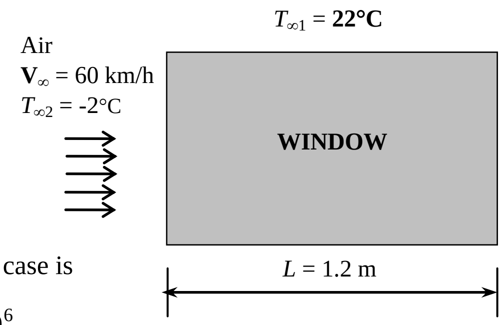
$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(90-70)\text{°F}}{0.0164\text{ h.°F/Btu}} = \mathbf{1220\ Btu/h}$$

7-93 Wind is blowing parallel to the walls of a house with windows. The rate of heat loss through the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air flow is turbulent because of the intense vibrations involved. **5** The minivan is modeled as a rectangular box. **6** Air is an ideal gas with constant properties. **6** The pressure of air is 1 atm.

Properties Assuming a film temperature of 5°C, the properties of air at 1 atm and this temperature are evaluated to be (Table A-15)

$$k = 0.02401 \text{ W/m.°C}$$
$$\upsilon = 1.382\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7350$$



Analysis Air flows along 1.2 m side. The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(60\times1000/3600) \text{ m/s}](1.2 \text{ m})}{1.382\times10^{-5} \text{ m}^2\text{/s}} = 1.447\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = (0.037\text{Re}_L^{0.8} - 871)\Pr^{1/3} = \left[0.037(1.447\times10^6)^{0.8} - 871\right](0.7350)^{1/3} = 2046$$

$$h = \frac{k}{L}Nu = \frac{0.02401 \text{ W/m.°C}}{1.2 \text{ m}}(2046) = 40.93 \text{ W/m}^2\text{.°C}$$

The thermal resistances are

$$A_s = 3(1.2 \text{ m})(1.5 \text{m}) = 5.4 \text{ m}^2$$

$$R_{conv,i} = \frac{1}{h_i A_s} = \frac{1}{(8 \text{ W/m}^2\text{.°C})(5.4 \text{ m}^2)} = 0.0231 \text{°C/W}$$

$$R_{cond} = \frac{L}{kA_s} = \frac{0.005 \text{ m}}{(0.78 \text{ W/m.°C})(5.4 \text{ m}^2)} = 0.0012 \text{°C/W}$$

$$R_{conv,o} = \frac{1}{h_o A_s} = \frac{1}{(40.93 \text{ W/m}^2\text{.°C})(5.4 \text{ m}^2)} = 0.0045 \text{°C/W}$$

Then the total thermal resistance and the heat transfer rate through the 3 windows become

$$R_{total} = R_{conv,i} + R_{cond} + R_{conv,o} = 0.0231 + 0.0012 + 0.0045 = 0.0288 \text{°C/W}$$

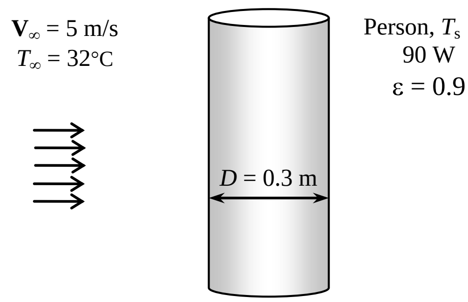
$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[22-(-2)]\text{°C}}{0.0288 \text{°C/W}} = \mathbf{833.3 \text{ W}}$$

7-94 A fan is blowing air over the entire body of a person. The average temperature of the outer surface of the person is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm. **4** The average human body can be treated as a 30-cm-diameter cylinder with an exposed surface area of 1.7 m^2.

Properties We assume the film temperature to be 35°C. The properties of air at 1 atm and this temperature are (Table A-15)

$$k = 0.02625 \text{ W/m.°C}$$
$$\upsilon = 1.655\times 10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7268$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{(5 \text{ m/s})(0.3 \text{ m})}{1.655\times 10^{-5} \text{ m}^2\text{/s}} = 9.063\times 10^4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\Pr^{1/3}}{\left[1+\left(0.4/\Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(9.063\times 10^4)^{0.5}(0.7268)^{1/3}}{\left[1+\left(0.4/0.7268\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{9.063\times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 203.6$$

Then

$$h = \frac{k}{D}Nu = \frac{0.02655 \text{ W/m.°C}}{0.3 \text{ m}}(203.6) = 18.02 \text{ W/m}^2\text{.°C}$$

Considering that there is heat generation in that person's body at a rate of 90 W and body gains heat by radiation from the surrounding surfaces, an energy balance can be written as

$$\dot{Q}_{generated} + \dot{Q}_{radiation} = \dot{Q}_{convection}$$

Substituting values with proper units and then application of trial & error method yields the average temperature of the outer surface of the person.

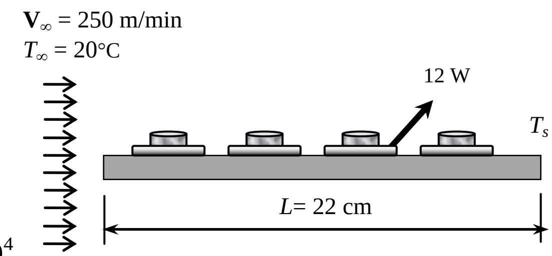
$$90 \text{ W} + \varepsilon A_s \sigma (T_{surr}{}^4 - T_s{}^4) = hA_s(T_s - T_\infty)$$
$$90 + (0.9)(1.7)(5.67\times 10^{-8})[(40+273)^4 - T_s{}^4] = (18.02)(1.7)[T_s - (32+273)]$$
$$T_s = \mathbf{309.2 K} = \mathbf{36.2°C}$$

7-95 The heat generated by four transistors mounted on a thin vertical plate is dissipated by air blown over the plate on both surfaces. The temperature of the aluminum plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** The entire plate is nearly isothermal. **5** The exposed surface area of the transistor is taken to be equal to its base area. **6** Air is an ideal gas with constant properties. **7** The pressure of air is 1 atm.

Properties Assuming a film temperature of 40°C, the properties of air are evaluated to be (Table A-15)

$$k = 0.02662\ \text{W/m.°C}$$
$$\upsilon = 1.702\times10^{-5}\ \text{m}^2/\text{s}$$
$$\Pr = 0.7255$$



Analysis The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(250/60)\ \text{m/s}](0.22\ \text{m})}{1.702\times10^{-5}\ \text{m}^2/\text{s}} = 5.386\times10^4$$

which is smaller than the critical Reynolds number. Thus we have laminar flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\Pr^{1/3} = 0.664(5.386\times10^4)^{0.5}(0.7255)^{1/3} = 138.5$$

$$h = \frac{k}{L}Nu = \frac{0.02662\ \text{W/m.°C}}{0.22\ \text{m}}(138.5) = 16.75\ \text{W/m}^2.\text{°C}$$

The temperature of aluminum plate then becomes

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 20\text{°C} + \frac{(4\times12)\ \text{W}}{(16.75\ \text{W/m}^2.\text{°C})[2(0.22\ \text{m})^2]} = \mathbf{50.0°C}$$

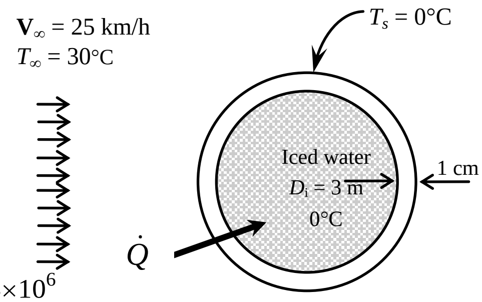
Discussion In reality, the heat transfer coefficient will be higher since the transistors will cause turbulence in the air.

7-96 A spherical tank used to store iced water is subjected to winds. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Thermal resistance of the tank is negligible. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\upsilon = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\mu_\infty = 1.872\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@\,0°C} = 1.729\times10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.7282$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(25\times1000/3600)\text{m/s}](3.02\text{ m})}{1.608\times10^{-5}\text{ m}^2\text{/s}} = 1.304\times10^6$$

The Nusselt number corresponding to this Reynolds number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(1.304\times10^6)^{0.5} + 0.06(1.304\times10^6)^{2/3}\right](0.7282)^{0.4}\left(\frac{1.872\times10^{-5}}{1.729\times10^{-5}}\right)^{1/4} = 1056$$

and $$h = \frac{k}{D}Nu = \frac{0.02588\text{ W/m.°C}}{3.02\text{ m}}(1056) = 9.05\text{ W/m}^2\text{.°C}$$

The rate of heat transfer to the iced water is

$$\dot{Q} = hA_s(T_s - T_\infty) = h(\pi D^2)(T_s - T_\infty) = (9.05\text{ W/m}^2\text{.°C})[\pi(3.02\text{ m})^2](30-0)\text{°C} = \mathbf{7779\ W}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (7.779\text{ kJ/s})(24\times3600\text{s}) = 672{,}079\text{ kJ}$$

Then the amount of ice that melts during this period becomes

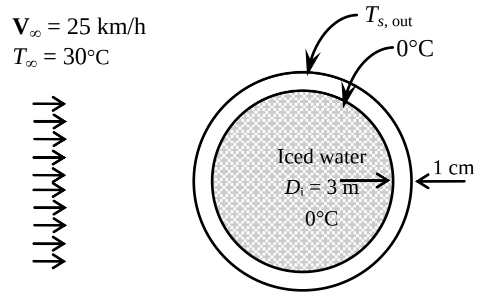
$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{672{,}079\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{2014\ \ kg}$$

7-97 A spherical tank used to store iced water is subjected to winds. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **7** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-15)

$k = 0.02588\ \text{W/m.°C}$	$\mu_{s,@0°C} = 1.729\times10^{-5}\ \text{kg/m.s}$
$\upsilon = 1.608\times10^{-5}\ \text{m}^2/\text{s}$	$\text{Pr} = 0.7282$
$\mu_\infty = 1.872\times10^{-5}\ \text{kg/m.s}$	



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_\infty D}{\upsilon} = \frac{[(25\times1000/3600)\text{m/s}](3.02\ \text{m})}{1.608\times10^{-5}\ \text{m}^2/\text{s}} = 1.304\times10^6$$

The Nusselt number corresponding to this Reynolds number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(1.304\times10^6)^{0.5} + 0.06(1.304\times10^6)^{2/3}\right](0.7282)^{0.4}\left(\frac{1.872\times10^{-5}}{1.729\times10^{-5}}\right)^{1/4} = 1056$$

and $$h = \frac{k}{D}Nu = \frac{0.02588\ \text{W/m.°C}}{3.02\ \text{m}}(1056) = 9.05\ \text{W/m}^2.\text{°C}$$

In steady operation, heat transfer through the tank by conduction is equal to the heat transfer from the outer surface of the tank by convection and radiation. Therefore,

$$\dot{Q} = \dot{Q}_{\text{through tank}} = \dot{Q}_{\text{from tank, conv+rad}}$$

$$\dot{Q} = \frac{T_{s,out} - T_{s,in}}{R_{sphere}} = h_o A_o (T_{surr} - T_{s,out}) + \varepsilon A_o \sigma (T_{surr}{}^4 - T_{s,out}{}^4)$$

where $$R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.51 - 1.50)\ \text{m}}{4\pi(15\ \text{W/m.°C})(1.51\text{m})(1.50\text{m})} = 2.342\times10^{-5}\ \text{°C/W}$$

$$A_o = \pi D^2 = \pi(3.02\ \text{m})^2 = 28.65\text{m}^2$$

Substituting,

$$\dot{Q} = \frac{T_{s,out} - 0°\text{C}}{2.34\times10^{-5}\ \text{°C/W}} = (9.05\ \text{W/m}^2.\text{°C})(28.65\text{m}^2)(30 - T_{s,out})\text{°C}$$
$$+ (0.9)(28.65\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(15 + 273\text{K})^4 - (T_{s,out} + 273\text{K})^4]$$

whose solution is

$$T_s = 0.23\text{°C} \text{ and } \dot{Q} = 9630\ \text{W} = \mathbf{9.63\ kW}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (9.63\ \text{kJ/s})(24\times3600\text{s}) = 832{,}032\ \text{kJ}$$

Then the amount of ice that melts during this period becomes

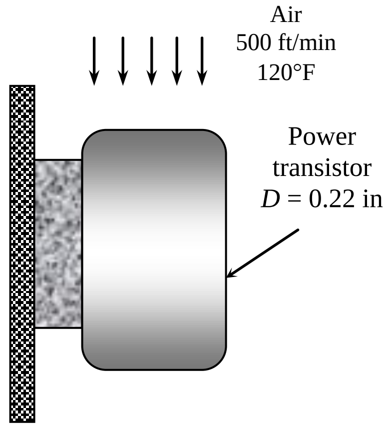
$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{832{,}032\ \text{kJ}}{333.7\ \text{kJ/kg}} = \mathbf{2493\ kg}$$

7-98E A cylindrical transistor mounted on a circuit board is cooled by air flowing over it. The maximum power rating of the transistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $T_f = (180+120)/2 = 150°\text{F}$ are (Table A-15)

$$k = 0.01646 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.210\times10^{-3} \text{ ft}^2/\text{s}$$
$$\Pr = 0.7188$$



Analysis The Reynolds number is

$$\text{Re} = \frac{\mathbf{V}_\infty D}{\upsilon} = \frac{(500/60 \text{ ft/s})(0.22/12 \text{ ft})}{0.210\times10^{-3} \text{ ft}^2/\text{s}} = 727.5$$

The Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(727.5)^{0.5}(0.7188)^{1/3}}{\left[1+(0.4/0.7188)^{2/3}\right]^{1/4}}\left[1+\left(\frac{727.5}{282{,}000}\right)^{5/8}\right]^{4/5} = 13.72$$

and $$h = \frac{k}{D}Nu = \frac{0.01646 \text{ Btu/h.ft.°F}}{(0.22/12 \text{ ft})}(13.72) = 12.32 \text{ Btu/h.ft}^2\text{.°F}$$

Then the amount of power this transistor can dissipate safely becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = h(\pi DL)(T_s - T_\infty)$$
$$= (12.32 \text{ Btu/h.ft}^2\text{.°F})\left[\pi(0.22/12 \text{ ft})(0.25/12 \text{ ft})\right](180-120)\text{°C}$$
$$= \mathbf{0.887 \text{ Btu/h} = 0.26 \text{ W}} \quad (1 \text{ W} = 3.412 \text{ Btu/h})$$