Chapter 8
INTERNAL FORCED CONVECTION

General Flow Analysis

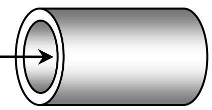
8-1C Liquids are usually transported in circular pipes because pipes with a circular cross-section can withstand large pressure differences between the inside and the outside without undergoing any distortion.

8-2C Reynolds number for flow in a circular tube of diameter D is expressed as

$$\mathrm{Re} = \frac{\mathbf{V}_m D}{\upsilon} \quad \text{where} \quad \mathbf{V}_\infty = \frac{\dot m}{\rho A_c} = \frac{\dot m}{\rho(\pi D^2/4)} = \frac{4\dot m}{\rho \pi D^2} \quad \text{and} \quad \upsilon = \frac{\mu}{\rho}$$

Substituting,

$$\mathrm{Re} = \frac{\mathbf{V}_m D}{\upsilon} = \frac{4\dot m D}{\rho \pi D^2(\mu/\rho)} = \frac{4\dot m}{\pi D \mu}$$



8-3C Engine oil requires a larger pump because of its much larger density.

8-4C The generally accepted value of the Reynolds number above which the flow in a smooth pipe is turbulent is 4000.

8-5C For flow through non-circular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the hydraulic diameter D_h defined as $D_h = \frac{4A_c}{p}$ where A_c is the cross-sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since $D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D$.

8-6C The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the *hydrodynamic entry region*, and the length of this region is called *hydrodynamic entry length*. The entry length is much longer in laminar flow than it is in turbulent flow. But at very low Reynolds numbers, L_h is very small (L_h = 1.2D at Re = 20).

8-7C The friction factor is highest at the tube inlet where the thickness of the boundary layer is zero, and decreases gradually to the fully developed value. The same is true for turbulent flow.

8-8C In turbulent flow, the tubes with rough surfaces have much higher friction factors than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the friction factor is negligible.

8-9C The friction factor f remains constant along the flow direction in the fully developed region in both laminar and turbulent flow.

8-10C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

8-11C The number of transfer units NTU is a measure of the heat transfer area and effectiveness of a heat transfer system. A small value of NTU (NTU < 5) indicates more opportunities for heat transfer whereas a large NTU value (NTU >5) indicates that heat transfer will not increase no matter how much we extend the length of the tube.

8-12C The logarithmic mean temperature difference $\Delta T_{\ln}$ is an exact representation of the average temperature difference between the fluid and the surface for the entire tube. It truly reflects the exponential decay of the local temperature difference. The error in using the arithmetic mean temperature increases to undesirable levels when ΔT_e differs from ΔT_i by great amounts. Therefore we should always use the logarithmic mean temperature.

8-13C The region of flow over which the thermal boundary layer develops and reaches the tube center is called the thermal entry region, and the length of this region is called the thermal entry length. The region in which the flow is both hydrodynamically (the velocity profile is fully developed and remains unchanged) and thermally (the dimensionless temperature profile remains unchanged) developed is called the fully developed region.

8-14C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

8-15C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

8-16C In the fully developed region of flow in a circular tube, the velocity profile will not change in the flow direction but the temperature profile may.

8-17C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\mathrm{Re}\,D$ and $L_t = 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 10D$ in turbulent flow. Noting that Pr >> 1 for oils, the thermal entry length is larger than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

8-18C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\mathrm{Re}\,D$ and $L_t = 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 10\,\mathrm{Re}$ in turbulent flow. Noting that Pr << 1 for liquid metals, the thermal entry length is smaller than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

8-19C In fluid flow, it is convenient to work with an average or mean velocity V_m and an average or mean temperature T_m which remain constant in incompressible flow when the cross-sectional area of the tube is constant. The V_m and T_m represent the velocity and temperature, respectively, at a cross section if all the particles were at the same velocity and temperature.

8-20C When the surface temperature of tube is constant, the appropriate temperature difference for use in the Newton's law of cooling is logarithmic mean temperature difference that can be expressed as

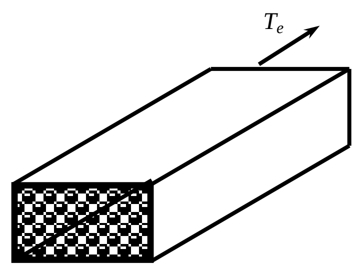
$$\Delta T_{\ln} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

8-21 Air flows inside a duct and it is cooled by water outside. The exit temperature of air and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the duct is constant. **3** The thermal resistance of the duct is negligible.

Properties The properties of air at the anticipated average temperature of 30°C are (Table A-15)

$$\rho = 1.164\ \mathrm{kg/m^3}$$
$$C_p = 1007\ \mathrm{J/kg.°C}$$

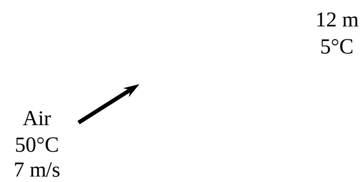


Analysis The mass flow rate of water is

$$\dot{m} = \rho A_c \mathbf{V}_m = \rho\left(\frac{\pi D^2}{4}\right)\mathbf{V}_m$$

$$= (1.164\,\text{kg/m}^3)\frac{\pi(0.2\,\text{m})^2}{4}(7\,\text{m/s}) = 0.256\,\text{kg/s}$$

$$A_s = \pi DL = \pi(0.2\,\text{m})(12\,\text{m}) = 7.54\,\text{m}^2$$



The exit temperature of air is determined from

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}C_p)} = 5-(5-50)e^{-\frac{(9.09)(7.54)}{(0.256)(1007)}} = \mathbf{8.74°C}$$

The logarithmic mean temperature difference and the rate of heat transfe r are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{8.74-50}{\ln\left(\frac{5-8.74}{5-50}\right)} = 16.59°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (85\,\text{W/m}^2.°\text{C})(7.54\,\text{m}^2)(16.59°\text{C}) = 10{,}6333.41\times10^4\ \text{W} = \mathbf{10{,}633\,W} \cong \mathbf{10.6\,kW}$$

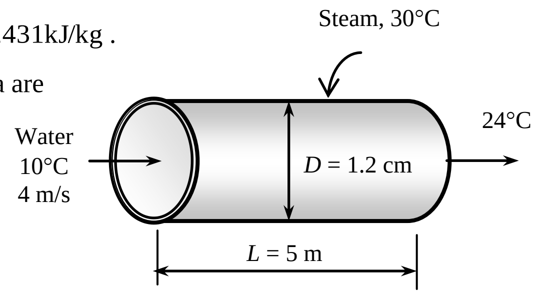
8-22 Steam is condensed by cooling water flowing inside copper tubes. The average heat transfer coefficient and the number of tubes needed are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible.

Properties The properties of water at the average temperature of (10+24)/2=17°C are (Table A-9)

$$\rho = 998.7 \text{ kg/m}^3$$
$$C_p = 4184.5 \text{ J/kg.°C}$$

Also, the heat of vaporization of water at 30°C is $h_{fg} = 2431 \text{kJ/kg}$.



Analysis The mass flow rate of water and the surface area are

$$\dot{m} = \rho A_c \mathbf{V}_m = \rho\left(\frac{\pi D^2}{4}\right)\mathbf{V}_m$$
$$= (998.7 \text{ kg/m}^3)\frac{\pi(0.012\text{m})^2}{4}(4\text{ m/s}) = 0.4518\text{kg/s}$$

The rate of heat transfer for one tube is

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.4518\text{kg/s})(4184.5\text{ J/kg.°C})(24-10\text{°C}) = 26{,}468\text{W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{24-10}{\ln\left(\dfrac{30-24}{30-10}\right)} = 11.63\text{°C}$$

$$A_s = \pi DL = \pi(0.012\text{m})(5\text{m}) = 0.1885\text{m}^2$$

The average heat transfer coefficient is determined from

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow h = \frac{\dot{Q}}{A_s\Delta T_{\ln}} = \frac{26{,}468\text{W}}{(0.1885\text{m}^2)(11.63\text{°C})}\left(\frac{1\text{ kW}}{1000\text{W}}\right) = \mathbf{12.1kW/m^2.°C}$$

The total rate of heat transfer is determined from

$$\dot{Q}_{total} = \dot{m}_{cond}h_{fg} = (0.15\text{ kg/s})(2431\text{kJ/kg}) = 364.65\text{ kW}$$

Then the number of tubes becomes

$$N_{tube} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{364{,}650\text{ W}}{26{,}468\text{ W}} = \mathbf{13.8}$$

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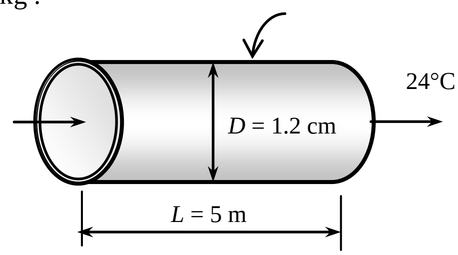

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The total rate of heat transfer is determined from

$$\dot{Q}_{total} = \dot{m}_{cond}h_{fg} = (0.60 \text{ kg/s})(2431 \text{kJ/kg}) = 1458.6 \text{ kW}$$

Then the number of tubes becomes

$$N_{tube} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{1{,}458{,}600 \text{W}}{26{,}468 \text{W}} = \mathbf{55.1}$$

8-24 Combustion gases passing through a tube are used to vaporize waste water. The tube length and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible. **4** Air properties are to be used for exhaust gases.

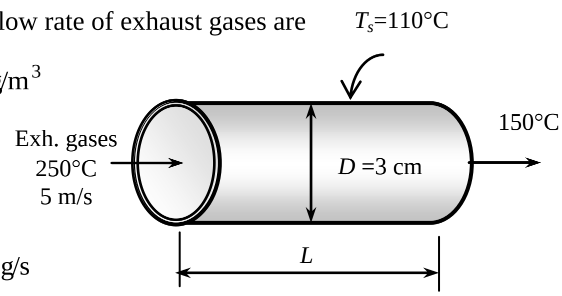
Properties The properties of air at the average temperature of (250+150)/2=200°C are (Table A-15)

$$C_p = 1023 \text{ J/kg.°C}$$

$$R = 0.287 \text{ kJ/kg.K}$$

Also, the heat of vaporization of water at 1 atm or 100°C is $h_{fg} = 2257 \text{ kJ/kg}$.

Analysis The density of air at the inlet and the mass flow rate of exhaust gases are



$$\rho = \frac{P}{RT} = \frac{115 \text{ kPa}}{(0.287 \text{ kJ/kg.K})(250 + 273 \text{K})} = 0.7662 \text{ kg/m}^3$$

$$\dot{m} = \rho A_c \mathbf{V}_m = \rho\left(\frac{\pi D^2}{4}\right)\mathbf{V}_m$$

$$= (0.7662 \text{ kg/m}^3)\frac{\pi(0.03\text{m})^2}{4}(5 \text{ m/s}) = 0.002708 \text{kg/s}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}C_p(T_i - T_e) = (0.002708 \text{kg/s})(1023 \text{J/kg.°C})(250 - 150\text{°C}) = 276.9 \text{ W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{150 - 250}{\ln\left(\frac{110 - 150}{110 - 250}\right)} = 79.82\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\ln}} = \frac{276.9 \text{ W}}{(120 \text{ W/m}^2.\text{°C})(79.82\text{°C})} = 0.02891 \text{m}^2$$

Then the tube length becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.02891 \text{m}^2}{\pi(0.03 \text{ m})} = 0.3067 \text{ m} = \mathbf{30.7 \text{ cm}}$$

The rate of evaporation of water is determined from

$$\dot{Q} = \dot{m}_{evap}h_{fg} \longrightarrow \dot{m}_{evap} = \frac{\dot{Q}}{h_{fg}} = \frac{(0.2769 \text{ kW})}{(2257 \text{ kJ/kg})} = 0.0001227 \text{kg/s} = \mathbf{0.442 \text{ kg/h}}$$

8-25 Combustion gases passing through a tube are used to vaporize waste water. The tube length and the rate of evaporation of water are to be determined.

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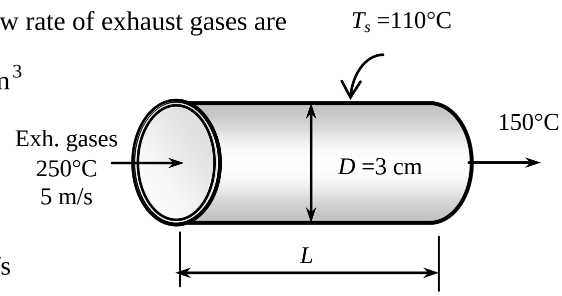


$$\rho = \frac{P}{RT} = \frac{115 \text{ kPa}}{(0.287 \text{ kJ/kg.K})(250+273 \text{ K})} = 0.7662 \text{ kg/m}^3$$

$$\dot{m} = \rho A_c \mathbf{V}_m = \rho\left(\frac{\pi D^2}{4}\right)\mathbf{V}_m$$

$$= (0.7662 \text{ kg/m}^3)\frac{\pi(0.03 \text{ m})^2}{4}(5 \text{ m/s}) = 0.002708 \text{ kg/s}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}C_p(T_i - T_e) = (0.002708 \text{ kg/s})(1023 \text{ J/kg.°C})(250-150\text{°C}) = 276.9 \text{ W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{150-250}{\ln\left(\frac{110-150}{110-250}\right)} = 79.82\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\ln}} = \frac{276.9 \text{ W}}{(60 \text{ W/m}^2.\text{°C})(79.82\text{°C})} = 0.05782 \text{ m}^2$$

Then the tube length becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.05782 \text{ m}^2}{\pi(0.03 \text{ m})} = 0.6135 \text{ m} = \mathbf{61.4 cm}$$

The rate of evaporation of water is determined from

$$\dot{Q} = \dot{m}_{evap}h_{fg} \longrightarrow \dot{m}_{evap} = \frac{\dot{Q}}{h_{fg}} = \frac{(0.2769 \text{ kW})}{(2257 \text{ kJ/kg})} = 0.0001227 \text{ kg/s} = \mathbf{0.442 kg/h}$$

Laminar and Turbulent Flow in Tubes

8-26C The friction factor for flow in a tube is proportional to the pressure drop. Since the pressure drop along the flow is directly related to the power requirements of the pump to maintain flow, the friction factor is also proportional to the power requirements. The applicable relations are

$$\Delta P = f\,\frac{L}{D}\frac{\rho V^2}{2} \quad \text{and} \quad \dot{W}_{\text{pump}} = \frac{\dot{m}\Delta P}{\rho}$$

8-27C The shear stress at the center of a circular tube during fully developed laminar flow is zero since the shear stress is proportional to the velocity gradient, which is zero at the tube center.

8-28C Yes, the shear stress at the surface of a tube during fully developed turbulent flow is maximum since the shear stress is proportional to the velocity gradient, which is maximum at the tube surface.

8-29C In fully developed flow in a circular pipe with negligible entrance effects, if the length of the pipe is doubled, the pressure drop will also *double* (the pressure drop is proportional to length).

8-30C Yes, the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2 since $\dot{V} = \mathbf{V}_{\text{ave}} A_c = (\mathbf{V}_{\max}/2) A_c$.

8-31C No, the average velocity in a circular pipe in fully developed laminar flow **cannot** be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). The mean velocity is $\mathbf{V}_{\max}/2$, but the velocity at $R/2$ is

$$\mathbf{V}(R/2) = \mathbf{V}_{\max}\left(1 - \frac{r^2}{R^2}\right)_{r=R/2} = \frac{3\mathbf{V}_{\max}}{4}$$

8-32C In fully developed laminar flow in a circular pipe, the pressure drop is given by

$$\Delta P = \frac{8\mu L \mathbf{V}_{\text{m}}}{R^2} = \frac{32\mu L \mathbf{V}_{\text{m}}}{D^2}$$

The mean velocity can be expressed in terms of the flow rate as $\mathbf{V}_{\text{m}} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4}$. Substituting,

$$\Delta P = \frac{8\mu L \mathbf{V}_{\text{m}}}{R^2} = \frac{32\mu L \mathbf{V}_{\text{m}}}{D^2} = \frac{32\mu L}{D^2}\frac{\dot{V}}{\pi D^2/4} = \frac{128\mu L\dot{V}}{\pi D^4}$$

Therefore, at constant flow rate and pipe length, the pressure drop is inversely proportional to the 4$^{\text{th}}$ power of diameter, and thus reducing the pipe diameter by half will increase the pressure drop **by a factor of 16** .

8-33C In fully developed laminar flow in a circular pipe, the pressure drop is given by

$$\Delta P = \frac{8\mu L \mathbf{V}_{\text{m}}}{R^2} = \frac{32\mu L \mathbf{V}_{\text{m}}}{D^2}$$

When the flow rate and thus mean velocity are held constant, the pressure drop becomes proportional to viscosity. Therefore, pressure drop will be **reduced by half** when the viscosity is reduced by half.

8-34C The tubes with rough surfaces have much higher heat transfer coefficients than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the heat transfer coefficient is negligible.

8-35 The flow rate through a specified water pipe is given. The pressure drop and the pumping power requirements are to be determined.

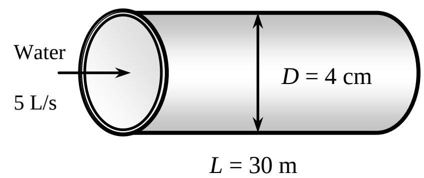
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be ρ = 999.1 kg/m³ and μ = 1.138×10⁻³ kg/m·s, respectively. The roughness of stainless steel is 0.002 mm (Table 8-3).

Analysis First we calculate the mean velocity and the Reynolds number to determine the flow regime:

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.005\,\text{m}^3/s}{\pi(0.04\,\text{m})^2/4} = 3.98\,\text{m}/s$$

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(999.1\,\text{kg/m}^3)(3.98\,\text{m/s})(0.04\,\text{m})}{1.138\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1.40\times10^5$$



which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{2\times10^{-6}\ \text{m}}{0.04\,\text{m}} = 5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{5\times10^{-5}}{3.7} + \frac{2.51}{1.40\times10^5\sqrt{f}}\right)$$

It gives f = 0.0171. Then the pressure drop and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0171\frac{30\,\text{m}}{0.04\,\text{m}}\frac{(999.1\,\text{kg/m}^3)(3.98\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = 101.5\ \text{kPa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.005\,\text{m}^3/s)(101.5\,\text{kPa})\left(\frac{1\,\text{kW}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.508\,kW}$$

Therefore, useful power input in the amount of 0.508 kW is needed to overcome the frictional losses in the pipe.

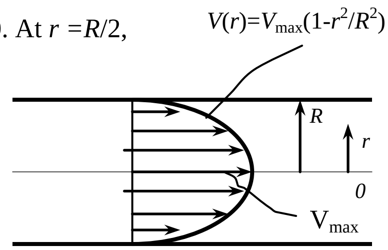
Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.0169, which is sufficiently close to 0.0171. Also, the friction factor corresponding to ε = 0 in this case is 0.0168, which indicates that stainless steel pipes can be assumed to be smooth with an error of about 2%. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

8-36 In fully developed laminar flow in a circular pipe, the velocity at $r = R/2$ is measured. The velocity at the center of the pipe ($r = 0$) is to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$\mathbf{V}(r) = \mathbf{V}_{max}\left(1 - \frac{r^2}{R^2}\right)$$



where $\mathbf{V}_{max}$ is the maximum velocity which occurs at pipe center, $r = 0$. At $r = R/2$,

$$\mathbf{V}(R/2) = \mathbf{V}_{max}\left(1 - \frac{(R/2)^2}{R^2}\right) = \mathbf{V}_{max}\left(1 - \frac{1}{4}\right) = \frac{3\mathbf{V}_{max}}{4}$$

Solving for $\mathbf{V}_{max}$ and substituting,

$$\mathbf{V}_{max} = \frac{4\mathbf{V}(R/2)}{3} = \frac{4(6\text{ m/s})}{3} = \mathbf{8\ m/s}$$

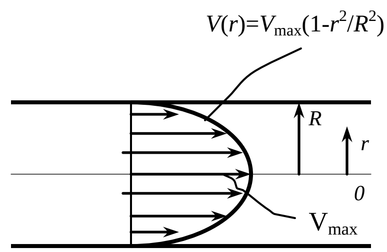
which is the velocity at the pipe center.

8-37 The velocity profile in fully developed laminar flow in a circular pipe is given. The mean and maximum velocities are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$\mathbf{V}(r) = \mathbf{V}_{max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$\mathbf{V}(r) = 4(1 - r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be $\mathbf{V}_{max}$ = 4 m/s. Then the mean velocity and volume flow rate become

$$\mathbf{V}_m = \frac{\mathbf{V}_{max}}{2} = \frac{4\text{ m/s}}{2} = \mathbf{2\ m/s}$$

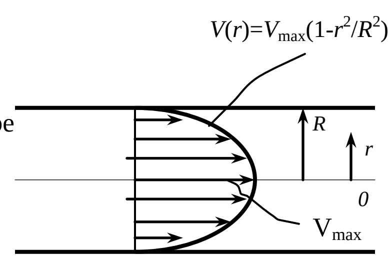
$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi R^2) = (2\text{ m/s})[\pi(0.02\text{m})^2] = \mathbf{0.00251 m^3/s}$$

8-38 The velocity profile in fully developed laminar flow in a circular pipe is given. The mean and maximum velocities are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by

$$\mathbf{V}(r) = \mathbf{V}_{max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$\mathbf{V}(r) = 4(1 - r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be $\mathbf{V}_{max}$ = 4 m/s. Then the mean velocity and volume flow rate become

$$\mathbf{V}_m = \frac{\mathbf{V}_{max}}{2} = \frac{4\text{ m/s}}{2} = \mathbf{2\ m/s}$$

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi R^2) = (2\text{ m/s})[\pi(0.05\text{m})^2] = \mathbf{0.0157 m^3/s}$$

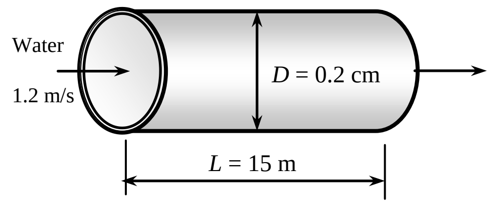
8-39 The average flow velocity in a pipe is given. The pressure drop and the pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be $\rho = 999.7$ kg/m^3 and $\mu = 1.307\times10^{-3}$ kg/m·s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number of the flow is

$$\text{Re} = \frac{\rho \mathbf{V}_m D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})(2\times10^{-3}\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1836$$



which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\text{Re}} = \frac{64}{1836} = 0.0349$$

$$\Delta P = f\,\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.0349\frac{15\,\text{m}}{0.002\,\text{m}}\frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = \mathbf{188\,kPa}$$

(*b*) The volume flow rate and the pumping power requirements are

$$\dot{V} = \mathbf{V}_m A_c = \mathbf{V}_m(\pi D^2/4) = (1.2\ \text{m/s})[\pi(0.002\text{m})^2/4] = 3.77\times10^{-6}\ \text{m}^3/s$$

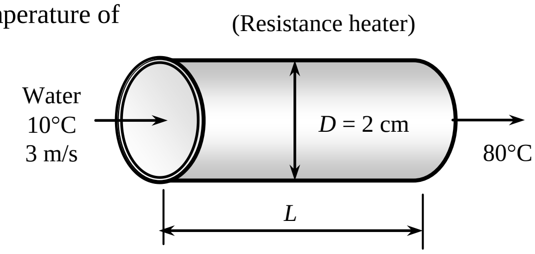
$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (3.77\times10^{-6}\ \text{m}^3/s)(188\,\text{kPa})\left(\frac{1000\,\text{W}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.71\,W}$$

Therefore, power input in the amount of 0.71 W is needed to overcome the frictional losses in the flow due to viscosity.

8-40 Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of (80+10) / 2 = 45°C are (Table A-9)



$$\rho = 990.1\ \text{kg/m}^3$$
$$k = 0.637\ \text{W/m.°C}$$
$$\upsilon = \mu/\rho = 0.602\times10^{-6}\ \text{m}^2/\text{s}$$
$$C_p = 4180\ \text{J/kg.°C}$$
$$\Pr = 3.91$$

Analysis The power rating of the resistance heater is

$$\dot{m} = \rho\dot{V} = (990.1\ \text{kg/m}^3)(0.008\ \text{m}^3/\text{min}) = 7.921\ \text{kg/min} = 0.132\ \text{kg/s}$$

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (0.132\ \text{kg/s})(4180\ \text{J/kg.°C})(80-10)\text{°C} = \mathbf{38{,}627\ W}$$

The velocity of water and the Reynolds number are

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{(8\times10^{-3}/60)\ \text{m}^3/\text{s}}{\pi(0.02\ \text{m})^2/4} = 0.4244\ \text{m/s}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(0.4244\ \text{m/s})(0.02\ \text{m})}{0.602\times10^{-6}\ \text{m}^2/\text{s}} = 14{,}101$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.02\ \text{m}) = 0.20\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\Pr^{0.4} = 0.023(14{,}101)^{0.8}(3.91)^{0.4} = 82.79$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.637\ \text{W/m.°C}}{0.02\ \text{m}}(82.79) = 2637\ \text{W/m}^2.\text{°C}$$

Then the inner surface temperature of the pipe at the exit becomes

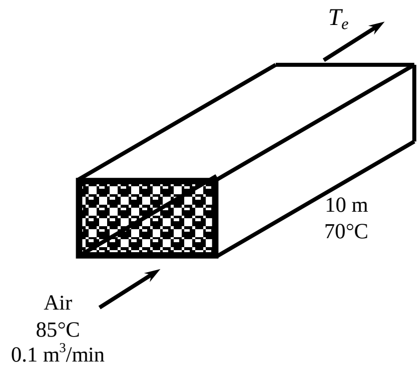
$$\dot{Q} = hA_s(T_{s,e} - T_e)$$
$$38{,}627\ \text{W} = (2637\ \text{W/m}^2.\text{°C})[\pi(0.02\ \text{m})(7\ \text{m})](T_s - 80)\text{°C}$$
$$T_{s,e} = \mathbf{113.3°C}$$

8-41 Flow of hot air through uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 80°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-15)

$$\rho = 0.9994\ \text{kg/m}^3$$
$$k = 0.02953\ \text{W/m.°C}$$
$$\upsilon = 2.097\times10^{-5}\ \text{m}^2\text{/s}$$
$$C_p = 1008\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7154$$



Analysis The characteristic length that is the hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{P} = \frac{4a^2}{4a} = a = 0.15\ \text{m}$$

$$\mathbf{V}_m = \frac{\dot{V}}{A_c} = \frac{0.10\ \text{m}^3\text{/s}}{(0.15\ \text{m})^2} = 4.444\ \text{m/s}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(4.444\ \text{m/s})(0.15\ \text{m})}{2.097\times10^{-5}\ \text{m}^2\text{/s}} = 31{,}791$$

which is greater than 10,0000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.15\ \text{m}) = 1.5\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(31{,}791)^{0.8}(0.7154)^{0.3} = 83.16$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.02953\ \text{W/m.°C}}{0.15\ \text{m}}(83.16) = 16.37\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of air,

$$A_s = 4aL = 4(0.15\ \text{m})(10\ \text{m}) = 6\ \text{m}^2$$
$$\dot{m} = \rho\dot{V} = (0.9994\ \text{kg/m}^3)(0.10\ \text{m}^3\text{/s}) = 0.09994\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}C_p)} = 70 - (70-85)e^{-\frac{(16.37)(6)}{(0.09994)(1008)}} = \mathbf{75.7°C}$$

Then the logarithmic mean temperature difference and the rate of heat loss from the air becomes

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{75.7-85}{\ln\left(\frac{70-75.7}{70-85}\right)} = 9.58°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (16.37\ \text{W/m}^2\text{.°C})(6\ \text{m}^2)(9.58°\text{C}) = \mathbf{941.1\ W}$$

Note that the temperature of air drops by almost 10°C as it flows in the duct as a result of heat loss.

8-42 "!PROBLEM 8-42"

```
"GIVEN"
T_i=85 "[C]"
L=10 "[m]"
side=0.15 "[m]"
"V_dot=0.10 [m^3/s], parameter to be varied"
T_s=70 "[C]"

"PROPERTIES"
Fluid$='air'
C_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=1/2*(T_i+T_e)

"ANALYSIS"
D_h=(4*A_c)/p
A_c=side^2
p=4*side
Vel=V_dot/A_c
Re=(Vel*D_h)/nu "The flow is turbulent"
L_t=10*D_h "The entry length is much shorter than the total length of the duct."
Nusselt=0.023*Re^0.8*Pr^0.3
h=k/D_h*Nusselt
A=4*side*L
m_dot=rho*V_dot
T_e=T_s-(T_s-T_i)*exp((-h*A)/(m_dot*C_p))
DELTAT_ln=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=h*A*DELTAT_ln
```

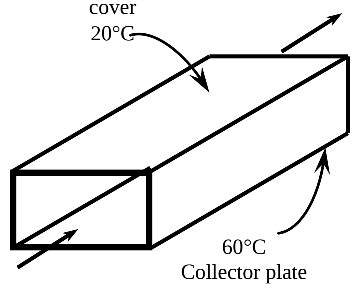
V [m^3/s]	**T_e [C]**	**Q [W]**
0.05	74.89	509
0.055	75	554.1
0.06	75.09	598.6
0.065	75.18	642.7
0.07	75.26	686.3
0.075	75.34	729.5
0.08	75.41	772.4
0.085	75.48	814.8
0.09	75.54	857
0.095	75.6	898.9
0.1	75.66	940.4
0.105	75.71	981.7
0.11	75.76	1023
0.115	75.81	1063
0.12	75.86	1104
0.125	75.9	1144
0.13	75.94	1184
0.135	75.98	1224
0.14	76.02	1264
0.145	76.06	1303
0.15	76.1	1343

8-43 Air enters the constant spacing between the glass cover and the plate of a solar collector. The net rate of heat transfer and the temperature rise of air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the spacing are smooth. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and estimated average temperature of 35°C are (Table A-15)

$$\rho = 1.146 \text{ kg/m}^3 \qquad C_p = 1007 \text{ J/kg.°C}$$
$$k = 0.02625 \text{ W/m.°C} \qquad \text{Pr} = 0.7268$$
$$\upsilon = 1.655\times10^{-5} \text{ m}^2/\text{s}$$



Analysis Mass flow rate, cross sectional area, hydraulic diameter, mean velocity of air and the Reynolds number are

$$\dot{m} = \rho\dot{V} = (1.146 \text{ kg/m}^3)(0.15 \text{ m}^3/\text{s}) = 0.1719 \text{ kg/s}$$

$$A_c = (1 \text{ m})(0.03 \text{ m}) = 0.03 \text{ m}^2$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.03 \text{ m}^2)}{2(1 \text{ m} + 0.03 \text{ m})} = 0.05825 \text{ m}$$

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.15 \text{ m}^3/\text{s}}{0.03 \text{ m}^2} = 5 \text{ m/s}$$

$$\text{Re} = \frac{V_m D_h}{\upsilon} = \frac{(5 \text{ m/s})(0.05825 \text{ m})}{1.655\times10^{-5} \text{ m}^2/\text{s}} = 17{,}606$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.05825 \text{m}) = 0.5825 \text{m}$$

which are much shorter than the total length of the collector. Therefore, we can assume fully developed turbulent flow in the entire collector, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} = 0.023(17{,}606)^{0.8}(0.7268)^{0.4} = 50.45$$

and $$h = \frac{k}{D_h}Nu = \frac{0.02625 \text{W/m.°C}}{0.05825 \text{m}}(50.45) = 22.73 \text{ W/m}^2.\text{°C}$$

The exit temperature of air can be calculated using the "average" surface temperature as

$$A_s = 2(5 \text{m})(1 \text{m}) = 10 \text{ m}^2$$

$$T_{s,ave} = \frac{60+20}{2} = 40\text{° C}$$

$$T_e = T_{s,ave} - (T_{s,ave} - T_i)\exp\left(-\frac{hA_s}{\dot{m}C_p}\right) = 40 - (40-30)\exp\left(-\frac{22.73\times10}{0.1718\times1007}\right) = 37.31\text{°C}$$

The temperature rise of air is

$$\Delta T = 37.3\text{°C} - 30\text{°C} = \mathbf{7.3\text{°C}}$$

The logarithmic mean temperature difference and the heat loss from the glass are

$$\Delta T_{\ln,glass} = \frac{T_e - T_i}{\ln\frac{T_s - T_e}{T_s - T_i}} = \frac{37.31-30}{\ln\frac{20-37.31}{20-30}} = 13.32\text{°C}$$

$$\dot{Q}_{glass} = hA_s\Delta T_{\ln} = (22.73 \text{W/m}^2.\text{°C})(5 \text{ m}^2)(13.32\text{°C}) = 1514 \text{ W}$$

The logarithmic mean temperature difference and the heat gain of the absorber are

$$\Delta T_{\ln,absorber} = \frac{T_e - T_i}{\ln\frac{T_s - T_e}{T_s - T_i}} = \frac{37.31-30}{\ln\frac{60-37.31}{60-30}} = 26.17\text{°C}$$

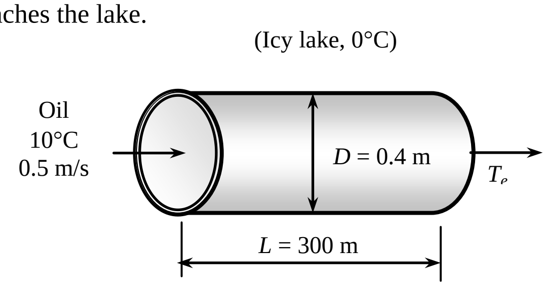
$$\dot{Q}_{absorber} = hA\Delta T_{\ln} = (22.73 \text{ W/m}^2.°\text{C})(5 \text{ m}^2)(26.17°\text{C}) = 2975 \text{ W}$$

Then the net rate of heat transfer becomes

$$\dot{Q}_{net} = 2975 - 1514 = \mathbf{1461\ W}$$

8-44 Oil flows through a pipeline that passes through icy waters of a lake. The exit temperature of the oil and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.



Properties The properties of oil at 10°C are (Table A-13)

$$\rho = 893.5 \text{ kg/m}^3, \qquad k = 0.146 \text{ W/m.°C}$$
$$\mu = 2.325 \text{ kg/m.s}, \qquad \upsilon = 2591\times10^{-6} \text{ m}^2\text{/s}$$
$$C_p = 1838 \text{ J/kg.°C}, \qquad \text{Pr} = 28750$$

Analysis (a) The Reynolds number in this case is

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(0.5 \text{ m/s})(0.4 \text{ m})}{2591\times10^{-6} \text{ m}^2\text{/s}} = 77.19$$

which is less than 2300. Therefore, the flow is laminar, and the thermal entry length is roughly

$$L_t = 0.05 \text{Re Pr} D = 0.05(77.19)(28750)(0.4 \text{ m}) = 44{,}384 \text{ m}$$

which is much longer than the total length of the pipe. Therefore, we assume thermally developing flow, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 3.66 + \frac{0.065(D/L)\text{Re Pr}}{1+0.04[(D/L)\text{Re Pr}]^{2/3}} = 3.66 + \frac{0.065\left(\frac{0.4 \text{ m}}{300 \text{ m}}\right)(77.19)(28{,}750)}{1+0.04\left[\left(\frac{0.4 \text{ m}}{300 \text{ m}}\right)(77.19)(28{,}750)\right]^{2/3}} = 24.47$$

and $$h = \frac{k}{D} Nu = \frac{0.146 \text{ W/m.°C}}{0.4 \text{ m}}(24.47) = 8.930 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of oil

$$A_s = \pi DL = \pi(0.4 \text{ m})(300 \text{ m}) = 377 \text{ m}^2$$

$$\dot{m} = \rho\dot{V} = \rho A_c \mathbf{V}_m = \rho\left(\frac{\pi D^2}{4}\right)\mathbf{V}_m = (893.5 \text{ kg/m}^3)\frac{\pi(0.4 \text{ m})^2}{4}(0.5 \text{ m/s}) = 56.14 \text{ kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}C_p)} = 0-(0-10)e^{-\frac{(8.930)(377)}{(56.14)(1838)}} = \mathbf{9.68°C}$$

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{9.68-10}{\ln\left(\frac{0-9.68}{0-10}\right)} = 9.84°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (8.930 \text{ W/m}^2\text{.°C})(377 \text{ m}^2)(9.84°\text{C}) = 3.31\times10^4 \text{ W} = \mathbf{3.31 kW}$$

The friction factor is

$$f = \frac{64}{\text{Re}} = \frac{64}{77.19} = 0.8291$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho\mathbf{V}_m^2}{2} = 0.8291\frac{300 \text{ m}}{0.4 \text{ m}}\frac{(893.5 \text{ kg/m}^3)(0.5 \text{ m/s})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right) = 69.54 \text{ kPa}$$

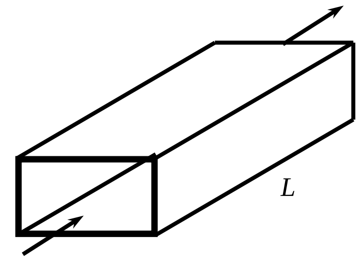
$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.0628 \text{ m}^3\text{/s})(69.54 \text{ kPa})\left(\frac{1 \text{ kW}}{1 \text{ kPa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{4.364 kW}$$

Discussion The power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

8-45 Laminar flow of a fluid through an isothermal square channel is considered. The change in the pressure drop and the rate of heat transfer are to be determined when the mean velocity is doubled.

Analysis The pressure drop of the fluid for laminar flow is expressed as

$$\Delta P_1 = f\,\frac{L}{D}\frac{\rho \mathbf{V}_m{}^2}{2} = \frac{64}{\mathrm{Re}}\frac{L}{D}\frac{\rho \mathbf{V}_m{}^2}{2} = \frac{64\upsilon}{\mathbf{V}_m D}\frac{L}{D}\frac{\rho \mathbf{V}_m{}^2}{2} = 32\mathbf{V}_m\,\frac{\upsilon L\rho}{D^2}$$



When the free-stream velocity of the fluid is doubled, the pressure drop becomes

$$\Delta P_2 = f\,\frac{L}{D}\frac{\rho (2\mathbf{V}_m)^2}{2} = \frac{64}{\mathrm{Re}}\frac{L}{D}\frac{\rho 4\mathbf{V}_m{}^2}{2} = \frac{64\upsilon}{2\mathbf{V}_m D}\frac{L}{D}\frac{\rho 4\mathbf{V}_m{}^2}{2} = 64\mathbf{V}_m\,\frac{\upsilon L\rho}{D^2}$$

Their ratio is

$$\frac{\Delta P_2}{\Delta P_1} = \frac{64}{32} = \mathbf{2}$$

The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA\Delta T_{\ln} = \frac{k}{D}\,NuA\Delta T_{\ln} = \frac{k}{D}1.86\left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.4} A\Delta T_{\ln}$$

$$= \frac{\mathbf{V}_m{}^{1/3}D^{1/3}}{\upsilon^{1/3}}\,\frac{k}{D}1.86\left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.4} A\Delta T_{\ln}$$

When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

$$\dot{Q}_2 = \frac{(2\mathbf{V}_m)^{1/3}D^{1/3}}{\upsilon^{1/3}}\,\frac{k}{D}1.86\left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.4} A\Delta T_{\ln}$$

Their ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\mathbf{V}_m)^{1/3}}{\mathbf{V}_m{}^{1/3}} = 2^{1/3} = \mathbf{1.26}$$

Therefore, doubling the velocity will double the pressure drop but it will increase the heat transfer rate by only 26%.

8-46 Turbulent flow of a fluid through an isothermal square channel is considered. The change in the pressure drop and the rate of heat transfer are to be determined when the free-stream velocity is doubled.

Analysis The pressure drop of the fluid for turbulent flow is expressed as

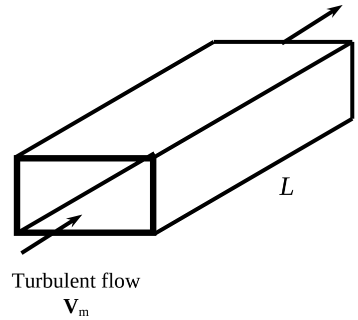
$$\Delta P_1 = f\,\frac{L}{D}\frac{\rho \mathbf{V}_m^{\,2}}{2} = 0.184\,\mathrm{Re}^{-0.2}\,\frac{L}{D}\frac{\rho \mathbf{V}_m^{\,2}}{2} = 0.184\,\frac{\mathbf{V}_m^{\,-0.2} D^{-0.2}}{\upsilon^{-0.2}}\,\frac{L}{D}\frac{\rho \mathbf{V}_m^{\,2}}{2}$$

$$= 0.092\mathbf{V}_m^{\,1.8}\left(\frac{D}{\upsilon}\right)^{-0.2}\frac{L\rho}{D}$$

When the free-stream velocity of the fluid is doubled, the pressure drop becomes

$$\Delta P_2 = f\,\frac{L}{D}\frac{\rho (2\mathbf{V}_m)^2}{2} = 0.184\,\mathrm{Re}^{-0.2}\,\frac{L}{D}\frac{\rho 4\mathbf{V}_m^{\,2}}{2} = 0.184\,\frac{(\mathbf{2V}_m)^{-0.2} D^{-0.2}}{\upsilon^{-0.2}}\,\frac{L}{D}\frac{\rho 4\mathbf{V}_m^{\,2}}{2}$$

$$= 0.368(2)^{-0.2}\mathbf{V}_m^{\,1.8}\left(\frac{D}{\upsilon}\right)^{-0.2}\frac{L\rho}{D}$$



Their ratio is

$$\frac{\Delta P_2}{\Delta P_1} = \frac{0.368(2)^{-0.2}\mathbf{V}_m^{\,1.8}}{0.092 V_m^{\,1.8}} = 4(2)^{-0.2} = \mathbf{3.48}$$

The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA\Delta T_{\ln} = \frac{k}{D} Nu A\Delta T_{\ln} = \frac{k}{D}0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\,A\Delta T_{\ln}$$

$$= 0.023\mathbf{V}_m^{\,0.8}\left(\frac{D}{\upsilon}\right)^{0.8}\frac{k}{D}\mathrm{Pr}^{1/3}\,A\Delta T_{\ln}$$

When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

$$\dot{Q}_2 = 0.023(2\mathbf{V}_m)^{0.8}\left(\frac{D}{\upsilon}\right)^{0.8}\frac{k}{D}\mathrm{Pr}^{1/3}\,A\Delta T_{\ln}$$

Their ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\mathbf{V}_m)^{0.8}}{\mathbf{V}_m^{\,0.8}} = 2^{0.8} = \mathbf{1.74}$$

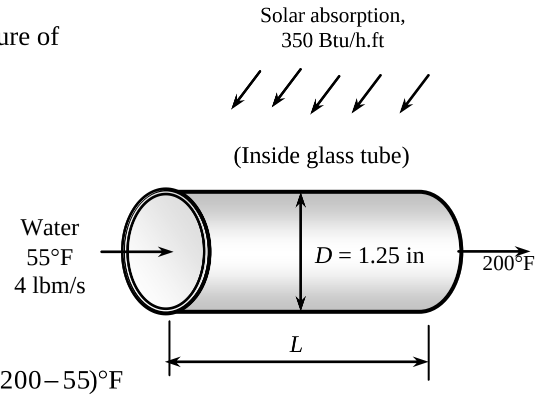
Therefore, doubling the velocity will increase the pressure drop 3.8 times but it will increase the heat transfer rate by only 74%.

8-47E Water is heated in a parabolic solar collector. The required length of parabolic collector and the surface temperature of the collector tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal resistance of the tube is negligible. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of (55+200)/2 = 127.5°F are (Table A-9E)

$$\rho = 61.59 \text{ lbm/ft}^3$$
$$k = 0.374 \text{ Btu/ft.°F}$$
$$\upsilon = \mu / \rho = 0.5683\times10^{-5} \text{ ft}^2\text{/s}$$
$$C_p = 0.999 \text{ Btu/lbm.°F}$$
$$\text{Pr} = 3.368$$



Analysis The total rate of heat transfer is

$$\dot{Q} = \dot{m}C_p(T_e - T_i) = (4 \text{ lbm/s})(0.999 \text{ Btu/lbm.°F})(200-55)\text{°F}$$
$$= 579.4 \text{ Btu/s} = 2.086\times10^6 \text{ Btu/h}$$

The length of the tube required is

$$L = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{2.086\times10^4 \text{ Btu/h}}{350 \text{ Btu/h.ft}} = \mathbf{5960\,ft}$$

The velocity of water and the Reynolds number are

$$\mathbf{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{4 \text{ lbm/s}}{(61.59 \text{ lbm/m}^3)\pi \dfrac{(1.25/12 \text{ ft})^2}{4}} = 7.621 \text{ ft/s}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(7.621 \text{ m/s})(1.25/12 \text{ ft})}{0.5683\times10^{-5} \text{ ft}^2\text{/s}} = 1.397\times10^5$$

which is greater than 10,000. Therefore, we can assume fully developed turbulent flow in the entire tube, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} = 0.023(1.397\times10^4)^{0.8}(3.368)^{0.4} = 488.4$$

The heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.374 \text{ Btu/h.ft.°F}}{1.25/12 \text{ ft}}(488.4) = 1754 \text{ Btu/h.ft}^2\text{.°F}$$

The heat flux on the tube is

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{2.086\times10^4 \text{ Btu/h}}{\pi(1.25/12 \text{ ft})(5960 \text{ ft})} = 1070 \text{ Btu/h.ft}^2$$

Then the surface temperature of the tube at the exit becomes

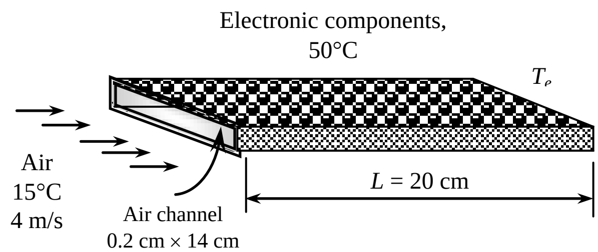
$$\dot{q} = h(T_s - T_e) \longrightarrow T_s = T_e + \frac{\dot{q}}{h} = 200\text{°F} + \frac{1070 \text{ Btu/h.ft}^2}{1754 \text{ Btu/h.ft}^2\text{.°F}} = \mathbf{200.6°F}$$

8-48 A circuit board is cooled by passing cool air through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Air is an ideal gas with constant properties. **5** The pressure of air in the channel is 1 atm.

Properties The properties of air at 1 atm and estimated average temperature of 25°C are (Table A-15)

$$\rho = 1.184\ \text{kg/m}^3$$
$$k = 0.02551\ \text{W/m.°C}$$
$$\upsilon = 1.562\times10^{-5}\ \text{m}^2/\text{s}$$
$$C_p = 1007\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7296$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002\ \text{m})(0.14\ \text{m}) = 0.00028\ \text{m}^2$$
$$A_s = (0.14\ \text{m})(0.2\ \text{m}) = 0.028\ \text{m}^2$$

To determine heat transfer coefficient, we first need to find the Reynolds number,

$$D_h = \frac{4A_c}{P} = \frac{4(0.00028\ \text{m}^2)}{2(0.002\ \text{m} + 0.14\ \text{m})} = 0.003944\ \text{m}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(4\ \text{m/s})(0.003944\ \text{m})}{1.562\times10^{-5}\ \text{m}^2/\text{s}} = 1010$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(1010)(0.7296)(0.003944\ \text{m}) = 0.1453\ \text{m} < 0.20\ \text{m}$$

Therefore, we have developing flow through most of the channel. However, we take the conservative approach and assume fully developed flow, and from Table 8-1 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h} Nu = \frac{0.02551\ \text{W/m.°C}}{0.003944\ \text{m}}(8.24) = 53.30\ \text{W/m}^2.\text{°C}$$

Also,

$$\dot{m} = \rho \mathbf{V} A_c = (1.184\ \text{kg/m}^3)(4\ \text{m/s})(0.00028\ \text{m}^2) = 0.001326\ \text{kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A_s = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$hA_s(T_s - T_e) = \dot{m}C_p(T_e - T_i)$$
$$(53.30\ \text{W/m}^2.\text{°C})(0.028\ \text{m}^2)(50\text{°C} - T_e) = (0.001326\ \text{kg/s})(1007\ \text{J/kg.°C})(T_e - 15\text{°C})$$
$$T_e = 33.5\text{°C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

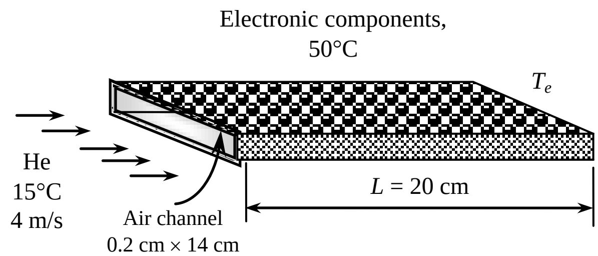
$$\dot{Q}_{\max} = \dot{m}C_p(T_e - T_i) = (0.001326\ \text{kg/s})(1007\ \text{J/kg.°C})(33.5 - 15\text{°C}) = \mathbf{24.7\ W}$$

8-49 A circuit board is cooled by passing cool helium gas through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Helium is an ideal gas. **5** The pressure of helium in the channel is 1 atm.

Properties The properties of helium at the estimated average temperature of 25°C are (Table A-16)

$$\rho = 0.1635\,\text{kg/m}^3$$
$$k = 0.1565\,\text{W/m.°C}$$
$$\upsilon = 1.233\times10^{-4}\ \text{m}^2\text{/s}$$
$$C_p = 5193\,\text{J/kg.°C}$$
$$\text{Pr} = 0.669$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002\,\text{m})(0.14\,\text{m}) = 0.00028\,\text{m}^2$$
$$A_s = (0.14\,\text{m})(0.2\,\text{m}) = 0.028\,\text{m}^2$$

To determine heat transfer coefficient, we need to first find the Reynolds number

$$D_h = \frac{4A_c}{P} = \frac{4(0.00028\,\text{m}^2)}{2(0.002\,\text{m}+0.14\,\text{m})} = 0.003944\,\text{m}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(4\,\text{m/s})(0.003944\,\text{m})}{1.233\times10^{-4}\ \text{m}^2\text{/s}} = 127.9$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(127.9)(0.669)(0.003944\,\text{m}) = 0.01687\,\text{m} << 0.20\,\text{m}$$

Therefore, the flow is fully developed flow, and from Table 8-3 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h}Nu = \frac{0.1565\,\text{W/m.°C}}{0.003944\,\text{m}}(8.24) = 327.0\ \text{W/m}^2\text{.°C}$$

Also,

$$\dot{m} = \rho \mathbf{V} A_c = (0.1635\,\text{kg/m}^3)(4\,\text{m/s})(0.00028\,\text{m}^2) = 0.0001831\,\text{kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A_s = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$\dot{m}C_p(T_e - T_i) = hA_s(T_s - T_e)$$
$$(0.0001831\,\text{kg/s})(5193\,\text{J/kg.°C})(T_e - 15°\text{C}) = (327.0\,\text{W/m}^2\text{.°C})(0.0568\,\text{m}^2)(50°\text{C} - T_e)$$
$$T_e = 46.7°\text{C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

$$\dot{Q}_{\max} = \dot{m}C_p(T_e - T_i) = (0.0001831\,\text{kg/s})(5193\,\text{J/kg.°C})(46.7 - 15°\text{C}) = \mathbf{30.2\,W}$$

8-50 "!PROBLEM 8-50"

```
"GIVEN"
L=0.20 "[m]"
width=0.14 "[m]"
height=0.002 "[m]"
T_i=15 "[C]"
Vel=4 "[m/s], parameter to be varied"
"T_s=50 [C], parameter to be varied"

"PROPERTIES"
Fluid$='air'
C_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=1/2*(T_i+T_e)

"ANALYSIS"
A_c=width*height
A=width*L
p=2*(width+height)
D_h=(4*A_c)/p
Re=(Vel*D_h)/nu "The flow is laminar"
L_t=0.05*Re*Pr*D_h
"Taking conservative approach and assuming fully developed laminar flow, from Table 8-1
we read"
Nusselt=8.24
h=k/D_h*Nusselt
m_dot=rho*Vel*A_c
Q_dot=h*A*(T_s-T_e)
Q_dot=m_dot*C_p*(T_e-T_i)
```

Vel [m/s]	**Q [W]**
1	9.453
2	16.09
3	20.96
4	24.67
5	27.57
6	29.91
7	31.82
8	33.41
9	34.76
10	35.92

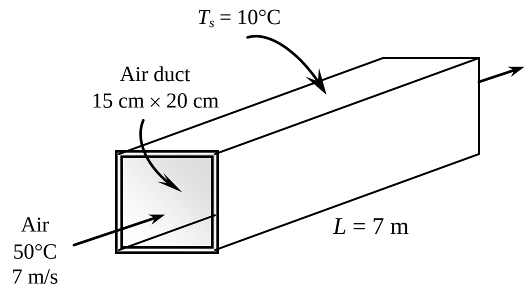
T_s [C]	**Q [W]**
30	10.59
35	14.12
40	17.64
45	21.15
50	24.67
55	28.18
60	31.68
65	35.18
70	38.68
75	42.17
80	45.65
85	49.13
90	52.6

8-51 Air enters a rectangular duct. The exit temperature of the air, the rate of heat transfer, and the fan power are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air in the duct is 1 atm.

Properties We assume the bulk mean temperature for air to be 40°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at this temperature and 1 atm are (Table A-15)

$$\rho = 1.127\ \text{kg/m}^3 \qquad C_p = 1007\ \text{J/kg.°C}$$
$$k = 0.02662\ \text{W/m.°C} \qquad \Pr = 0.7255$$
$$\upsilon = 1.702\times10^{-5}\ \text{m}^2/\text{s}$$



Analysis (*a*) The hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{P} = \frac{4(0.15\ \text{m})(0.20\ \text{m})}{2\ (0.15\ \text{m})+(0.20\ \text{m})} = 0.1714\ \text{m}$$

$$\text{Re} = \frac{\mathbf{V}_m D_h}{\upsilon} = \frac{(7\ \text{m/s})(0.1714\ \text{m})}{1.702\times10^{-5}\ \text{m}^2/\text{s}} = 70{,}525$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.1714\text{m}) = 1.714\text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\Pr^{0.3} = 0.023(70{,}525)^{0.8}(0.7255)^{0.3} = 158.0$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.02662\ \text{W/m.°C}}{0.1714\text{m}}(158.0) = 24.53\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air

$$A_s = 2\times7[(0.15\,\text{m})+(0.20\,\text{m})] = 4.9\,\text{m}^2$$
$$A_c = (0.15\,\text{m})(0.20\text{m}) = 0.03\text{m}^2$$
$$\dot{m} = \rho VA_c = (1.127\ \text{kg/m}^3)(7\ \text{m/s})(0.03\text{m}^2) = 0.2367\text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}C_p)} = 10-(10-50)e^{-\frac{(24.53)(4.9)}{(0.2367)(1007)}} = \mathbf{34.2°C}$$

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the air are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s-T_e}{T_s-T_i}\right)} = \frac{34.2-50}{\ln\left(\frac{10-34.2}{10-50}\right)} = 31.42\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (24.53\ \text{W/m}^2.\text{°C})(4.9\ \text{m}^2)(31.42\text{°C}) = \mathbf{3776\ W}$$

(*c*) The friction factor, the pressure drop, and then the fan power can be determined for the case of fully developed turbulent flow to be

$$f = 0.184\,\text{Re}^{-0.2} = 0.184(70{,}525)^{-0.2} = 0.01973$$

$$\Delta P = f\frac{L}{D}\frac{\rho \mathbf{V}_m^2}{2} = 0.01973\frac{(7\ \text{m})}{(0.1714\text{m})}\frac{(1.127\ \text{kg/m}^3)(7\ \text{m/s})^2}{2} = 22.25\ \text{N/m}^2$$

$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.2367\ \text{kg/s})(22.25\ \text{N/m}^2)}{1.127\ \text{kg/m}^3} = \mathbf{4.67\ W}$$

8-52 "!PROBLEM 8-52"

```
"GIVEN"
L=7 "[m]"
width=0.15 "[m]"
height=0.20 "[m]"
T_i=50 "[C]"
"Vel=7 [m/s], parameter to be varied"
T_s=10 "[C]"

"PROPERTIES"
Fluid$='air'
C_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=1/2*(T_i+T_e)

"ANALYSIS"
"(a)"
A_c=width*height
p=2*(width+height)
D_h=(4*A_c)/p
Re=(Vel*D_h)/nu "The flow is turbulent"
L_t=10*D_h "The entry length is much shorter than the total length of the duct."
Nusselt=0.023*Re^0.8*Pr^0.3
h=k/D_h*Nusselt
A=2*L*(width+height)
m_dot=rho*Vel*A_c
T_e=T_s-(T_s-T_i)*exp((-h*A)/(m_dot*C_p))
"(b)"
DELTAT_ln=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=h*A*DELTAT_ln
"(c)"
f=0.184*Re^(-0.2)
DELTAP=f*L/D_h*(rho*Vel^2)/2
W_dot_pump=(m_dot*DELTAP)/rho
```

Vel [m/s]	**T_e [C]**	**Q [W]**	**W_{pump} [W]**
1	29.01	715.6	0.02012
1.5	30.14	1014	0.06255
2	30.92	1297	0.1399
2.5	31.51	1570	0.2611
3	31.99	1833	0.4348
3.5	32.39	2090	0.6692
4	32.73	2341	0.9722
4.5	33.03	2587	1.352
5	33.29	2829	1.815
5.5	33.53	3066	2.369
6	33.75	3300	3.022
6.5	33.94	3531	3.781
7	34.12	3759	4.652
7.5	34.29	3984	5.642
8	34.44	4207	6.759
8.5	34.59	4427	8.008
9	34.72	4646	9.397
9.5	34.85	4862	10.93
10	34.97	5076	12.62