Chapter 9
NATURAL CONVECTION

Physical Mechanisms of Natural Convection

9-1C Natural convection is the mode of heat transfer that occurs between a solid and a fluid which moves under the influence of natural means. Natural convection differs from forced convection in that fluid motion in natural convection is caused by natural effects such as buoyancy.

9-2C The convection heat transfer coefficient is usually higher in forced convection because of the higher fluid velocities involved.

9-3C The hot boiled egg in a spacecraft will cool faster when the spacecraft is on the ground since there is no gravity in space, and thus there will be no natural convection currents which is due to the buoyancy force.

9-4C The upward force exerted by a fluid on a body completely or partially immersed in it is called the buoyancy or "lifting" force. The buoyancy force is proportional to the density of the medium. Therefore, the buoyancy force is the largest in mercury, followed by in water, air, and the evacuated chamber. Note that in an evacuated chamber there will be no buoyancy force because of absence of any fluid in the medium.

9-5C The buoyancy force is proportional to the density of the medium, and thus is larger in sea water than it is in fresh water. Therefore, the hull of a ship will sink deeper in fresh water because of the smaller buoyancy force acting upwards.

9-6C A spring scale measures the "weight" force acting on it, and the person will weigh less in water because of the upward buoyancy force acting on the person's body.

9-7C The greater the volume expansion coefficient, the greater the change in density with temperature, the greater the buoyancy force, and thus the greater the natural convection currents.

9-8C There cannot be any natural convection heat transfer in a medium that experiences no change in volume with temperature.

9-9C The lines on an interferometer photograph represent isotherms (constant temperature lines) for a gas, which correspond to the lines of constant density. Closely packed lines on a photograph represent a large temperature gradient.

9-10C The Grashof number represents the ratio of the buoyancy force to the viscous force acting on a fluid. The inertial forces in Reynolds number is replaced by the buoyancy forces in Grashof number.

9-11 The volume expansion coefficient is defined as $\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$. For an ideal gas, $P = \rho RT$ or $\rho = \frac{P}{RT}$, and thus $\beta = -\frac{1}{\rho}\left(\frac{\partial (P/RT)}{\partial T}\right)_P = \frac{-1}{\rho}\left(\frac{-P}{RT^2}\right) = \frac{1}{\rho T}\left(\frac{P}{RT}\right) = \frac{1}{\rho T}(\rho) = \frac{1}{T}$

Natural Convection Over Surfaces

9-12C Rayleigh number is the product of the Grashof and Prandtl numbers.

9-13C A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr^{1/4}}$.

9-14C No, a hot surface will cool slower when facing down since the warmer air in this position cannot rise and escape easily.

9-15C The heat flux will be higher at the bottom of the plate since the thickness of the boundary layer which is a measure of thermal resistance is the lowest there.

9-16 A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the pipe is constant.

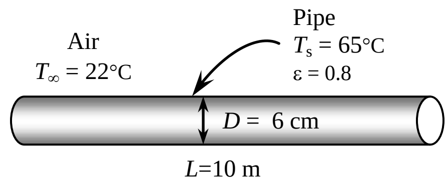
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (65+22)/2 = 43.5°C$ are (Table A-15)

$$k = 0.02688 \text{ W/m.°C}$$

$$\upsilon = 1.735\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7245$$

$$\beta = \frac{1}{T_f} = \frac{1}{(43.5+273)\text{K}} = 0.00316 \text{K}^{-1}$$



Analysis (*a*) The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.06$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00316 \text{ K}^{-1})(65-22 \text{ K})(0.06 \text{ m})^3}{(1.735\times10^{-5} \text{ m}^2\text{/s})^2}(0.7245) = 692{,}805$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(692{,}805)^{1/6}}{\left[1+\left(0.559/0.7245\right)^{9/16}\right]^{8/27}}\right\}^2 = 13.15$$

$$h = \frac{k}{D}Nu = \frac{0.02688 \text{ W/m.°C}}{0.06 \text{ m}}(13.15) = 5.893 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.06 \text{ m})(10 \text{ m}) = 1.885 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.893 \text{ W/m}^2\text{.°C})(1.885 \text{ m}^2)(65-22)\text{°C} = \mathbf{477.6 \text{ W}}$$

(*b*) The radiation heat loss from the pipe is

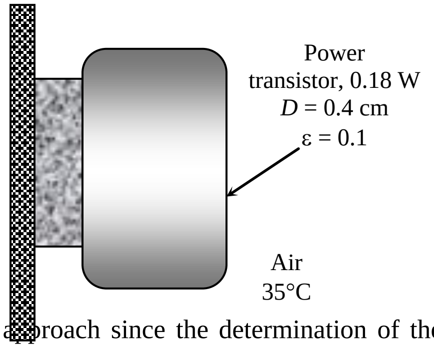
$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) = (0.8)(1.885 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(65+273 \text{K})^4 - (22+273 \text{K})^4\right] = \mathbf{468.4 \text{ W}}$$

9-17 A power transistor mounted on the wall dissipates 0.18 W. The surface temperature of the transistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Any heat transfer from the base surface is disregarded. **4** The local atmospheric pressure is 1 atm. **5** Air properties are evaluated at 100°C.

Properties The properties of air at 1 atm and the given film temperature of 100°C are (Table A-15)

$$k = 0.03095\ \text{W/m.°C}$$
$$\upsilon = 2.306\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7111$$
$$\beta = \frac{1}{T_f} = \frac{1}{(100+273)\ \text{K}} = 0.00268\ \text{K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 165°C for the evaluation of *h*. This is the surface temperature that will give a film temperature of 100°C. We will check the accuracy of this guess later and repeat the calculations if necessary.

The transistor loses heat through its cylindrical surface as well as its top surface. For convenience, we take the heat transfer coefficient at the top surface of the transistor to be the same as that of its side surface. (The alternative is to treat the top surface as a vertical plate, but this will double the amount of calculations without providing much improvement in accuracy since the area of the top surface is much smaller and it is circular in shape instead of being rectangular). The characteristic length in this case is the outer diameter of the transistor, $L_c = D = 0.004\ \text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.00268\ \text{K}^{-1})(165-35\ \text{K})(0.004\ \text{m})^3}{(2.306\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7111) = 292.6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(292.6)^{1/6}}{\left[1+(0.559/0.7111)^{9/16}\right]^{8/27}}\right\}^2 = 2.039$$

$$h = \frac{k}{D}Nu = \frac{0.03095\ \text{W/m.°C}}{0.004\ \text{m}}(2.039) = 15.78\ \text{W/m}^2.\text{°C}$$

$$A_s = \pi DL + \pi D^2/4 = \pi(0.004\ \text{m})(0.0045\ \text{m}) + \pi(0.004\ \text{m})^2/4 = 0.0000691\ \text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$0.18\ \text{W} = (15.8\ \text{W/m}^2.\text{°C})(0.0000691\ \text{m}^2)(T_s - 35)\ \text{°C}$$
$$+(0.1)(0.0000691\ \text{m}^2)(5.67\times10^{-8})\left[(T_s+273)^4 - (25+273\ \text{K})^4\right]$$
$$\longrightarrow T_s = 187\text{°C}$$

which is relatively close to the assumed value of 165°C. To improve the accuracy of the result, we repeat the Rayleigh number calculation at new surface temperature of 187°C and determine the surface temperature to be

T_s = **183°C**

Discussion W evaluated the air properties again at 100°C when repeating the calculation at the new surface temperature. It can be shown that the effect of this on the calculated surface temperature is less than 1°C.

9-18 "!PROBLEM 9-18"

"GIVEN"
Q_dot=0.18 "[W]"

```
"T_infinity=35 [C], parameter to be varied"
L=0.0045 "[m]"
D=0.004 "[m]"
epsilon=0.1
T_surr=T_infinity-10 "[C]"

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s+T_infinity)
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"
g=9.807 "[m/s^2], gravitational acceleration"

"ANALYSIS"
delta=D
Ra=(g*beta*(T_s-T_infinity)*delta^3)/nu^2*Pr
Nusselt=(0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27))^2
h=k/delta*Nusselt
A=pi*D*L+pi*D^2/4
Q_dot=h*A*(T_s-T_infinity)+epsilon*A*sigma*((T_s+273)^4-(T_surr+273)^4)
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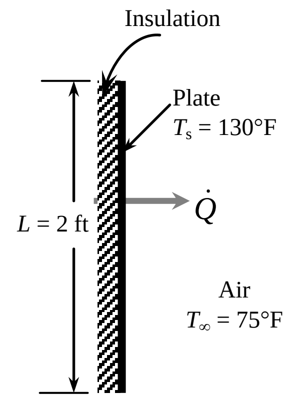
T_∞ [C]	T_s [C]
10	159.9
12	161.8
14	163.7
16	165.6
18	167.5
20	169.4
22	171.3
24	173.2
26	175.1
28	177
30	178.9
32	180.7
34	182.6
36	184.5
38	186.4
40	188.2

9-19E A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (130+75)/2 = 102.5°F are (Table A-15)



$$k = 0.01535 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.1823\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7256$$
$$\beta = \frac{1}{T_f} = \frac{1}{(102.5+460)\text{R}} = 0.001778\text{R}^{-1}$$

Analysis (*a*) When the plate is vertical, the characteristic length is the height of the plate. $L_c = L = 2$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001778\text{R}^{-1})(130-75\text{ R})(2\text{ ft})^3}{(0.1823\times10^{-3}\text{ ft}^2\text{/s})^2}(0.7256) = 5.503\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.503\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7256}\right)^{9/16}\right]^{8/27}}\right\}^2 = 102.6$$

$$h = \frac{k}{L}Nu = \frac{0.01535 \text{ Btu/h.ft.°F}}{2\text{ ft}}(102.6) = 0.7869 \text{ Btu/h.ft}^2\text{.°F}$$
$$A_s = L^2 = (2\text{ ft})^2 = 4\text{ ft}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (0.7869 \text{ Btu/h.ft}^2\text{.°F})(4\text{ ft}^2)(130-75)\text{°C} = \mathbf{173.1\ Btu/h}$$

(*b*) When the plate is horizontal with hot surface facing up, the characteristic length is determined from

$$L_s = \frac{A_s}{P} = \frac{L^2}{4L} = \frac{L}{4} = \frac{2\text{ ft}}{4} = 0.5\text{ ft}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001778\text{R}^{-1})(130-75\text{ R})(0.5\text{ ft})^3}{(0.1823\times10^{-3}\text{ ft}^2\text{/s})^2}(0.7256) = 8.598\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(8.598\times10^6)^{1/4} = 29.24$$

$$h = \frac{k}{L_c}Nu = \frac{0.01535 \text{ Btu/h.ft.°F}}{0.5\text{ ft}}(29.24) = 0.8975 \text{ Btu/h.ft}^2\text{.°F}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (0.8975 \text{ Btu/h.ft}^2\text{.°F})(4\text{ ft}^2)(130-75)\text{°C} = \mathbf{197.4\ Btu/h}$$

(*c*) When the plate is horizontal with hot surface facing down, the characteristic length is again $\delta = 0.5$ ft and the Rayleigh number is $Ra = 8.598\times10^6$. Then,

$$Nu = 0.27Ra^{1/4^2} = 0.27(8.598\times10^6)^{1/4} = 14.62$$

$$h = \frac{k}{L_c}Nu = \frac{0.01535 \text{ Btu/h.ft.°F}}{0.5\text{ ft}}(14.62) = 0.4487 \text{ Btu/h.ft}^2\text{.°F}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (0.4487\,\text{Btu/h.ft}^2.°\text{F})(4\,\text{ft}^2)(130 - 75)°\text{C} = \textbf{98.7 Btu/h}$$

9-20E "!PROBLEM 9-20E"

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"GIVEN"
L=2 "[ft]"
T_infinity=75 "[F]"
"T_s=130 [F], parameter to be varied"

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=14.7)
mu=Viscosity(Fluid$, T=T_film)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
beta=1/(T_film+460)
T_film=1/2*(T_s+T_infinity)
g=32.2 "[ft/s^2], gravitational acceleration"

"ANALYSIS"
"(a), plate is vertical"
delta_a=L
Ra_a=(g*beta*(T_s-T_infinity)*delta_a^3)/nu^2*Pr
Nusselt_a=0.59*Ra_a^0.25
h_a=k/delta_a*Nusselt_a
A=L^2
Q_dot_a=h_a*A*(T_s-T_infinity)
"(b), plate is horizontal with hot surface facing up"
delta_b=A/p
p=4*L
Ra_b=(g*beta*(T_s-T_infinity)*delta_b^3)/nu^2*Pr
Nusselt_b=0.54*Ra_b^0.25
h_b=k/delta_b*Nusselt_b
Q_dot_b=h_b*A*(T_s-T_infinity)
"(c), plate is horizontal with hot surface facing down"
delta_c=delta_b
Ra_c=Ra_b
Nusselt_c=0.27*Ra_c^0.25
h_c=k/delta_c*Nusselt_c
Q_dot_c=h_c*A*(T_s-T_infinity)
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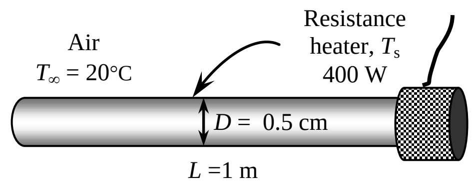
T_s [F]	Q_a [Btu/h]	Q_b [Btu/h]	Q_c [Btu/h]
80	7.714	9.985	4.993
85	18.32	23.72	11.86
90	30.38	39.32	19.66
95	43.47	56.26	28.13
100	57.37	74.26	37.13
105	71.97	93.15	46.58
110	87.15	112.8	56.4
115	102.8	133.1	66.56
120	119	154	77.02
125	135.6	175.5	87.75
130	152.5	197.4	98.72
135	169.9	219.9	109.9
140	187.5	242.7	121.3
145	205.4	265.9	132.9
150	223.7	289.5	144.7
155	242.1	313.4	156.7
160	260.9	337.7	168.8
165	279.9	362.2	181.1
170	299.1	387.1	193.5
175	318.5	412.2	206.1
180	338.1	437.6	218.8

9-21 A cylindrical resistance heater is placed horizontally in a fluid. The outer surface temperature of the resistance wire is to be determined for two different fluids.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer by radiation is ignored. **5** Properties are evaluated at 500°C for air and 40°C for water.

Properties The properties of air at 1 atm and 500°C are (Table A-15)

$$k = 0.05572\ \text{W/m.°C}$$
$$\upsilon = 7.804\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.6986$$
$$\beta = \frac{1}{T_f} = \frac{1}{(500+273)\text{K}} = 0.001294\ \text{K}^{-1}$$



The properties of water at 40°C are

$$k = 0.631\ \text{W/m.°C}$$
$$\upsilon = \mu/\rho = 0.6582\times10^{-6}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 4.32$$
$$\beta = 0.000377\ \text{K}^{-1}$$

Analysis (*a*) The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 1200°C for the calculation of *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005\text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{m/s}^2)(0.001294\text{K}^{-1})(1200-20)\text{°C}(0.005\text{m})^3}{(7.804\times10^{-5}\ \text{m}^2/\text{s})^2}(0.6986) = 214.7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(214.7)^{1/6}}{\left[1+(0.559/0.6986)^{9/16}\right]^{8/27}}\right\}^2 = 1.919$$

$$h = \frac{k}{D}Nu = \frac{0.05572\text{W/m.°C}}{0.005\text{m}}(1.919) = 21.38\ \text{W/m}^2.\text{°C}$$

$$A_s = \pi DL = \pi(0.005\text{m})(1\text{m}) = 0.01571\text{m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$400\text{W} = (21.38\text{W/m}^2.\text{°C})(0.01571\text{m}^2)(T_s - 20)\text{°C}$$
$$T_s = \mathbf{1211°C}$$

which is sufficiently close to the assumed value of 1200°C used in the evaluation of *h*, and thus it is not necessary to repeat calculations.

(*b*) For the case of water, we "guess" the surface temperature to be 40°C. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005\text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{m/s}^2)(0.000377\text{K}^{-1})(40-20\,\text{K})(0.005\text{m})^3}{(0.6582\times10^{-6}\ \text{m}^2/\text{s})^2}(4.32) = 92{,}197$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(92{,}197)^{1/6}}{\left[1+(0.559/4.32)^{9/16}\right]^{8/27}}\right\}^2 = 8.986$$

$$h = \frac{k}{D} Nu = \frac{0.631\,\text{W/m.°C}}{0.005\,\text{m}}(8.986) = 1134\,\text{W/m}^2.\text{°C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$400\,\text{W} = (1134\,\text{W/m}^2.\text{°C})(0.01571\text{m}^2)(T_s - 20)\text{°C}$$
$$T_s = \mathbf{42.5°C}$$

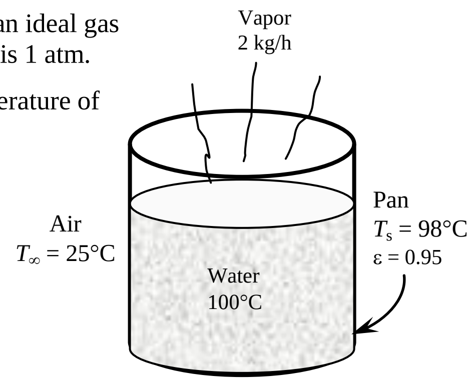
which is sufficiently close to the assumed value of 40°C in the evaluation of the properties and *h*. The film temperature in this case is $(T_s+T_\infty)/2$ = (42.5+20)/2 =31.3°C, which is close to the value of 40°C used in the evaluation of the properties.

9-22 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (98+25)/2 = 61.5$°C are (Table A-15)

$$k = 0.02819 \text{ W/m.°C}$$
$$\upsilon = 1.910\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7198$$
$$\beta = \frac{1}{T_f} = \frac{1}{(61.5+273)\text{K}} = 0.00299 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is the height of the pan, $L_c = L = 0.12$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.910\times10^{-5} \text{ m}^2\text{/s})^2}(0.7198) = 7.299\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.299\times10^6/0.7198)^{1/4}} = 0.07443 < 0.25 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$

Therefore,

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.299\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7198}\right)^{9/16}\right]^{8/27}}\right\}^2 = 28.60$$

$$h = \frac{k}{L}Nu = \frac{0.02819 \text{ W/m.°C}}{0.12 \text{ m}}(28.60) = 6.720 \text{ W/m}^2\text{.°C}$$
$$A_s = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.720 \text{ W/m}^2\text{.°C})(0.09425 \text{ m}^2)(98-25)\text{°C} = \mathbf{46.2\ W}$$

(*b*) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_s^4 - T_{surr}^4)$$
$$= (0.95)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{56.1\ W}$$

(*c*) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{fg} = (2/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 1.254 \text{ kW} = 1254 \text{ W}$$

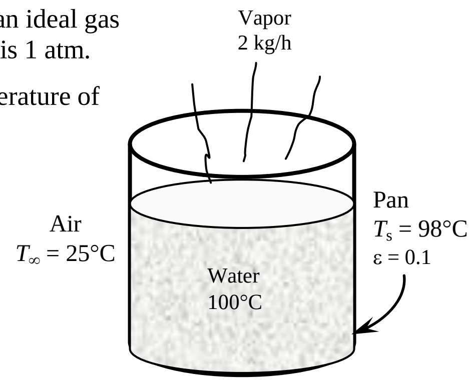
Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

$$f = \frac{46.2+56.1}{1254} = 0.082 = \mathbf{8.2\%}$$

9-23 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (98+25)/2 = 61.5°C$ are (Table A-15)



$$k = 0.02819 \text{ W/m.°C}$$
$$\upsilon = 1.910\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7198$$
$$\beta = \frac{1}{T_f} = \frac{1}{(61.5+273)\text{K}} = 0.00299 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the pan, $L_c = L = 0.12$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.910\times10^{-5} \text{ m}^2\text{/s})^2}(0.7198) = 7.299\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.299\times10^6/0.7198)^{1/4}} = 0.07443 < 0.25 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$

Therefore,

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.299\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7198}\right)^{9/16}\right]^{8/27}}\right\}^2 = 28.60$$

$$h = \frac{k}{L}Nu = \frac{0.02819 \text{ W/m.°C}}{0.12 \text{ m}}(28.60) = 6.720 \text{ W/m}^2\text{.°C}$$
$$A_s = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.720 \text{ W/m}^2\text{.°C})(0.09425 \text{ m}^2)(98-25)\text{°C} = \mathbf{46.2\ W}$$

(b) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_s^4 - T_{surr}^4)$$
$$= (0.10)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{5.9\ W}$$

(c) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{fg} = (2/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 1.254 \text{ kW} = 1254 \text{ W}$$

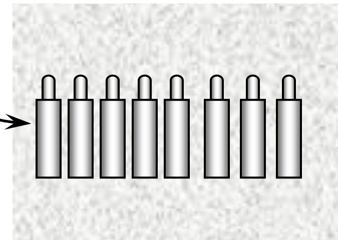
Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

$$f = \frac{46.2+5.9}{1254} = 0.042 = \mathbf{4.2\%}$$

9-24 Some cans move slowly in a hot water container made of sheet metal. The rate of heat loss from the four side surfaces of the container and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (55+20)/2 = 37.5°C$ are (Table A-15)



$$k = 0.02644 \text{ W/m.°C}$$
$$\upsilon = 1.678\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7261$$
$$\beta = \frac{1}{T_f} = \frac{1}{(37.5+273)\text{K}} = 0.003221 \text{K}^{-1}$$

Analysis The characteristic length in this case is the height of the bath, $L_c = L = 0.5 \text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003221 \text{K}^{-1})(55-20 \text{ K})(0.5 \text{ m})^3}{(1.678\times10^{-5} \text{ m}^2\text{/s})^2}(0.7261) = 3.565\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.565\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7261}\right)^{9/16}\right]^{8/27}}\right\}^2 = 89.84$$

$$h = \frac{k}{L}Nu = \frac{0.02644 \text{ W/m.°C}}{0.5 \text{ m}}(89.84) = 4.75 \text{ W/m}^2\text{.°C}$$
$$A_s = 2\left[(0.5 \text{ m})(1 \text{ m}) + (0.5 \text{ m})(3.5 \text{ m})\right] = 4.5 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (4.75 \text{ W/m}^2\text{.°C})(4.5 \text{ m}^2)(55-20)\text{°C} = 748.1 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\ 4} - T_{surr}^{\ 4})$$
$$= (0.7)(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(55+273 \text{K})^4 - (20+273 \text{K})^4\right] = 750.9 \text{ W}$$

Then the total rate of heat loss becomes

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural\\convection}} + \dot{Q}_{rad} = 748.1 + 750.9 = \mathbf{1499 \text{ W}}$$

The amount and cost of the heat loss during one year is

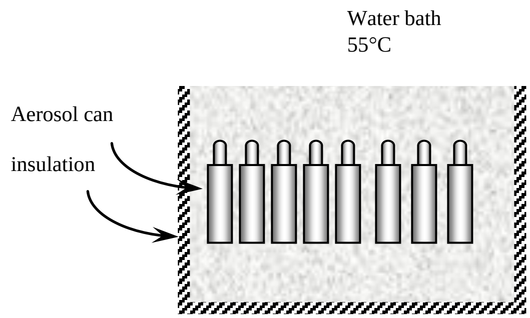
$$Q_{total} = \dot{Q}_{total}\Delta t = (1.499 \text{ kW})(8760 \text{ h}) = 13{,}131 \text{ kWh}$$
$$\text{Cost} = (13{,}131 \text{ kWh})(\$0.085/\text{ kWh}) = \mathbf{\$1116}$$

9-25 Some cans move slowly in a hot water container made of sheet metal. It is proposed to insulate the side and bottom surfaces of the container for $350. The simple payback period of the insulation to pay for itself from the energy it saves is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties Insulation will drop the outer surface temperature to a value close to the ambient temperature. The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature, which is unknown. We assume the surface temperature to be 26°C. The properties of air at the anticipated film temperature of (26+20)/2=23°C are (Table A-15)

$$k = 0.02536\,\text{W/m.°C}$$
$$\upsilon = 1.543\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7301$$
$$\beta = \frac{1}{T_f} = \frac{1}{(23+273)\text{K}} = 0.00338\,\text{K}^{-1}$$



Analysis We start the solution process by "guessing" the outer surface temperature to be 26 °C. We will check the accuracy of this guess later and repeat the calculations if necessary with a better guess based on the results obtained. The characteristic length in this case is the height of the tank, $L_c = L = 0.5\,\text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.00338\,\text{K}^{-1})(26-20\,\text{K})(0.5\,\text{m})^3}{(1.543\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7301) = 7.622\times10^7$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.622\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7301}\right)^{9/16}\right]^{8/27}}\right\}^2 = 56.53$$

$$h = \frac{k}{L}Nu = \frac{0.02536\,\text{W/m.°C}}{0.5\,\text{m}}(56.53) = 2.868\,\text{W/m}^2.\text{°C}$$
$$A_s = 2[(0.5\,\text{m})(1.10\,\text{m}) + (0.5\,\text{m})(3.60\,\text{m})] = 4.7\,\text{m}^2$$

Then the total rate of heat loss from the outer surface of the insulated tank by convection and radiation becomes

$$\begin{aligned}\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} &= hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)\\ &= (2.868\,\text{W/m}^2.\text{°C})(4.7\,\text{m}^2)(26-20)\text{°C}\\ &\quad + (0.1)(4.7\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(26+273\,\text{K})^4 - (20+273\,\text{K})^4]\\ &= 97.5\,\text{W}\end{aligned}$$

In steady operation, the heat lost by the side surfaces of the tank must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. The second conditions requires the surface temperature to be

$$\dot{Q} = \dot{Q}_{insulation} = kA_s\frac{T_{\text{tank}} - T_s}{L} \rightarrow 97.5\,\text{W} = (0.035\,\text{W/m.°C})(4.7\,\text{m}^2)\frac{(55-T_s)\text{°C}}{0.05\,\text{m}}$$

It gives T_s = 25.38°C, which is very close to the assumed temperature, 26°C. Therefore, there is no need to repeat the calculations.

The total amount of heat loss and its cost during one year are

$$Q_{total} = \dot{Q}_{total}\Delta t = (97.5\,\text{W})(8760\,\text{h}) = 853.7\,\text{kWh}$$

$$\text{Cost} = (853.7\text{ kWh})(\$0.085/\text{kWh}) = \$72.6$$

Then money saved during a one-year period due to insulation becomes

$$\text{Money saved} = \text{Cost}_{\substack{without\\insulation}} - \text{Cost}_{\substack{with\\insulation}} = \$1116 - \$72.6 = \$1043$$

where $1116 is obtained from the solution of Problem 9-24.

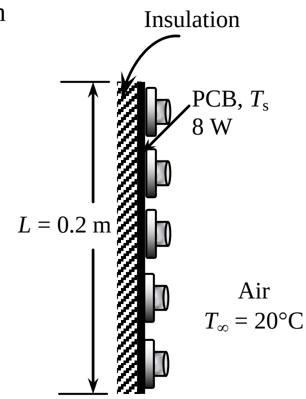
The insulation will pay for itself in

$$\text{Payback period} = \frac{\text{Cost}}{\text{Money saved}} = \frac{\$350}{\$1043/\,yr} = \mathbf{0.3354\,yr = 122\,days}$$

Discussion We would definitely recommend the installation of insulation in this case.

9-26 A printed circuit board (PCB) is placed in a room. The average temperature of the hot surface of the board is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** The heat loss from the back surface of the board is negligible.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (45+20)/2 = 32.5$°C are (Table A-15)

$$k = 0.02607\ \text{W/m.°C}$$
$$\upsilon = 1.631\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7275$$
$$\beta = \frac{1}{T_f} = \frac{1}{(32.5+273)\text{K}} = 0.003273\ \text{K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown

(*a*) **Vertical PCB** . We start the solution process by "guessing" the surface temperature to be 45°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the PCB, $L_c = L = 0.2$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003273\ \text{K}^{-1})(45-20\ \text{K})(0.2\ \text{m})^3}{(1.631\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7275) = 1.756\times10^7$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.756\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7275}\right)^{9/16}\right]^{8/27}}\right\}^2 = 36.78$$

$$h = \frac{k}{L}Nu = \frac{0.02607\ \text{W/m.°C}}{0.2\ \text{m}}(36.78) = 4.794\ \text{W/m}^2.\text{°C}$$

$$A_s = (0.15\ \text{m})(0.2\ \text{m}) = 0.03\ \text{m}^2$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$8\ \text{W} = (4.794\ \text{W/m}^2.\text{°C})(0.03\ \text{m}^2)(T_s - 20)\text{°C} + (0.8)(0.03\ \text{m}^2)(5.67\times10^{-8})\left[(T_s+273)^4 - (20+273\ \text{K})^4\right]$$

Its solution is

$$T_s = \mathbf{46.6°\ C}$$

which is sufficiently close to the assumed value of 45°C for the evaluation of the properties and *h*.

(*b*) **Horizontal, hot surface facing up** Again we assume the surface temperature to be 45 °C and use the properties evaluated above. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(0.20\ \text{m})(0.15\ \text{m})}{2(0.2\ \text{m}+0.15\ \text{m})} = 0.0429\ \text{m}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003273\ \text{K}^{-1})(45-20\ \text{K})(0.0429\ \text{m})^3}{(1.631\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7275) = 1.728\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.728\times10^5)^{1/4} = 11.01$$

$$h = \frac{k}{L_c}Nu = \frac{0.02607\ \text{W/m.°C}}{0.0429\ \text{m}}(11.01) = 6.696\ \text{W/m}^2.\text{°C}$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4)$$

$$8\,\text{W} = (6.696\,\text{W/m}^2.°\text{C})(0.03\,\text{m}^2)(T_s - 20)°\text{C} + (0.8)(0.03\,\text{m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273\,\text{K})^4]$$

Its solution is

$$T_s = \mathbf{42.6°C}$$

which is sufficiently close to the assumed value of 45°C in the evaluation of the properties and h.

(c) **Horizontal, hot surface facing down** This time we expect the surface temperature to be higher, and assume the surface temperature to be 50°C. We will check this assumption after obtaining result and repeat calculations with a better assumption, if necessary. The properties of air at the film temperature of

$$T_f = \frac{T_s + T_\infty}{2} = \frac{50 + 20}{2} = 35°\text{C} \text{ are (Table A-15)}$$

$$k = 0.02625\,\text{W/m.°C}$$

$$\upsilon = 1.655\times10^{-5}\ \text{m}^2/\text{s}$$

$$\text{Pr} = 0.7268$$

$$\beta = \frac{1}{T_f} = \frac{1}{(35 + 273)\text{K}} = 0.003247\text{K}^{-1}$$

The characteristic length in this case is, from part (b), L_c = 0.0429 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003247\text{K}^{-1})(50 - 20\,\text{K})(0.0429\,\text{m})^3}{(1.655\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7268) = 166{,}379$$

$$Nu = 0.27Ra^{1/4} = 0.27(166{,}379)^{1/4} = 5.453$$

$$h = \frac{k}{L_c}Nu = \frac{0.02625\,\text{W/m.°C}}{0.0429\,\text{m}}(5.453) = 3.340\,\text{W/m}^2.°\text{C}$$

Considering both natural convection and radiation heat loses

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4)$$

$$8\ \text{W} = (3.340\,\text{W/m}^2.°\text{C})(0.03\,\text{m}^2)(T_s - 20)°\text{C} + (0.8)(0.03\,\text{m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273\,\text{K})^4]$$

Its solution is

$$T_s = \mathbf{50.7°C}$$

which is very close to the assumed value. Therefore, there is no need to repeat calculations.

9-27 "!PROBLEM 9-27"

```
"GIVEN"
L=0.2 "[m]"
w=0.15 "[m]"
"T_infinity=20 [C], parameter to be varied"
Q_dot=8 "[W]"
epsilon=0.8 "parameter to be varied"
T_surr=T_infinity

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s_a+T_infinity)
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"
g=9.807 "[m/s^2], gravitational acceleration"

"ANALYSIS"
"(a), plate is vertical"
delta_a=L
Ra_a=(g*beta*(T_s_a-T_infinity)*delta_a^3)/nu^2*Pr
Nusselt_a=0.59*Ra_a^0.25
h_a=k/delta_a*Nusselt_a
A=w*L
Q_dot=h_a*A*(T_s_a-T_infinity)+epsilon*A*sigma*((T_s_a+273)^4-(T_surr+273)^4)
"(b), plate is horizontal with hot surface facing up"
delta_b=A/p
p=2*(w+L)
Ra_b=(g*beta*(T_s_b-T_infinity)*delta_b^3)/nu^2*Pr
Nusselt_b=0.54*Ra_b^0.25
h_b=k/delta_b*Nusselt_b
Q_dot=h_b*A*(T_s_b-T_infinity)+epsilon*A*sigma*((T_s_b+273)^4-(T_surr+273)^4)
"(c), plate is horizontal with hot surface facing down"
delta_c=delta_b
Ra_c=Ra_b
Nusselt_c=0.27*Ra_c^0.25
h_c=k/delta_c*Nusselt_c
Q_dot=h_c*A*(T_s_c-T_infinity)+epsilon*A*sigma*((T_s_c+273)^4-(T_surr+273)^4)
```

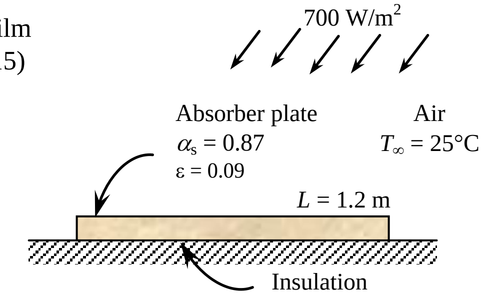
T_∞ [F]	$T_{s,a}$ [C]	$T_{s,b}$ [C]	$T_{s,c}$ [C]
5	32.54	28.93	38.29
7	34.34	30.79	39.97
9	36.14	32.65	41.66
11	37.95	34.51	43.35
13	39.75	36.36	45.04
15	41.55	38.22	46.73
17	43.35	40.07	48.42
19	45.15	41.92	50.12
21	46.95	43.78	51.81
23	48.75	45.63	53.51
25	50.55	47.48	55.21
27	52.35	49.33	56.91
29	54.16	51.19	58.62
31	55.96	53.04	60.32
33	57.76	54.89	62.03
35	59.56	56.74	63.74

9-28 Absorber plates whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (115+25)/2 = 70°C$ are (Table A-15)

$$k = 0.02881 \text{ W/m.°C}$$
$$\upsilon = 1.995\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915 \text{K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 115°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is $L_c = \frac{A_s}{p} = \frac{(1.2\text{ m})(0.8\text{ m})}{2(1.2\text{ m}+0.8\text{ m})} = 0.24\text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.002915\text{ K}^{-1})(115-25\text{ K})(0.24\text{ m})^3}{(1.995\times10^{-5}\text{ m}^2\text{/s})^2}(0.7177) = 6.414\times10^7$$
$$Nu = 0.54Ra^{1/4} = 0.54(6.414\times10^7)^{1/4} = 48.33$$
$$h = \frac{k}{L_c}Nu = \frac{0.02881\text{ W/m.°C}}{0.24\text{ m}}(48.33) = 5.801\text{ W/m}^2\text{.°C}$$
$$A_s = (0.8\text{ m})(1.2\text{ m}) = 0.96\text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A_s = (0.87)(700\text{ W/m}^2)(0.96\text{ m}^2) = 584.6\text{ W}$$
$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^4 - T_{sky}^4)$$
$$584.6\text{ W} = (5.801\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)\text{°C} + (0.09)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $T_s = \mathbf{115.6°C}$

which is identical to the assumed value. Therefore there is no need to repeat calculations.

If the absorber plate is made of ordinary aluminum which has a solar absorptivity of 0.28 and an emissivity of 0.07, the rate of solar gain becomes

$$\dot{Q} = \alpha\dot{q}A_s = (0.28)(700\text{ W/m}^2)(0.96\text{ m}^2) = 188.2\text{ W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience,

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^4 - T_{sky}^4)$$
$$188.2\text{ W} = (5.801\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)\text{°C} + (0.07)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is T_s = **55.2°C**

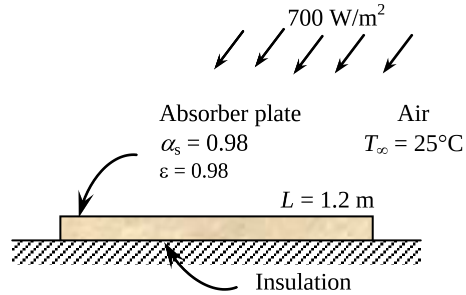
Repeating the calculations at the new film temperature of 40°C, we obtain

h = 4.524 W/m^2.°C and T_s = **62.8°C**

9-29 An absorber plate whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (70+25)/2 = 47.5°C are (Table A-15)



$$k = 0.02717 \text{ W/m.°C}$$
$$\upsilon = 1.774\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7235$$
$$\beta = \frac{1}{T_f} = \frac{1}{(47.5+273)\text{K}} = 0.00312 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 70°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is $L_c = \frac{A_s}{p} = \frac{(1.2\text{ m})(0.8\text{ m})}{2(1.2\text{ m}+0.8\text{ m})} = 0.24\text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.00312\text{ K}^{-1})(70-25\text{ K})(0.24\text{ m})^3}{(1.774\times10^{-5}\text{ m}^2\text{/s})^2}(0.7235) = 4.379\times10^7$$

$$Nu = 0.54Ra^{1/4} = 0.54(4.379\times10^7)^{1/4} = 43.93$$

$$h = \frac{k}{L_c}Nu = \frac{0.02717\text{ W/m.°C}}{0.24\text{ m}}(43.93) = 4.973\text{ W/m}^2\text{.°C}$$

$$A_s = (0.8\text{ m})(1.2\text{ m}) = 0.96\text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A_s = (0.98)(700\text{ W/m}^2)(0.96\text{ m}^2) = 658.6\text{ W}$$

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$658.6\text{ W} = (4.973\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)\text{°C} + (0.98)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $T_s = \mathbf{73.5°C}$

which is close to the assumed value. Therefore there is no need to repeat calculations.

For a white painted absorber plate, the solar absorptivity is 0.26 and the emissivity is 0.90. Then the rate of solar gain becomes

$$\dot{Q} = \alpha\dot{q}A_s = (0.26)(700\text{ W/m}^2)(0.96\text{ m}^2) = 174.7\text{ W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience (actually, we should calculate the new h using data at a lower temperature, and iterating if necessary for better accuracy),

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$174.7\text{ W} = (4.973\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)\text{°C} + (0.90)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

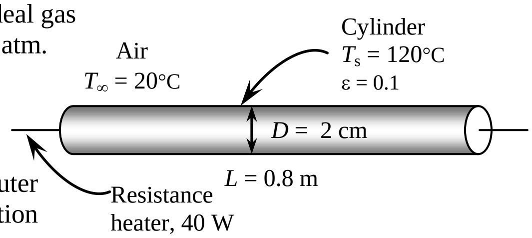
Its solution is $T_s = \mathbf{35.0°C}$

Discussion If we recalculated the h using air properties at 30°C, we would obtain

h = 3.47 W/m².°C and T_s = 36.6°C.

9-30 A resistance heater is placed along the centerline of a horizontal cylinder whose two circular side surfaces are well insulated. The natural convection heat transfer coefficient and whether the radiation effect is negligible are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Analysis The heat transfer surface area of the cylinder is

$$A = \pi DL = \pi(0.02\,\text{m})(0.8\,\text{m}) = 0.05027\,\text{m}^2$$

Noting that in steady operation the heat dissipated from the outer surface must equal to the electric power consumed, and radiation is negligible, the convection heat transfer is determined to be

$$\dot{Q} = hA_s(T_s - T_\infty) \rightarrow \ h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{40\,\text{W}}{(0.05027\,\text{m}^2)(120-20)^\circ\text{C}} = \mathbf{7.96\,W/m^2.^\circ C}$$

The radiation heat loss from the cylinder is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4) \\ &= (0.1)(0.05027\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(120+273\,\text{K})^4 - (20+273\,\text{K})^4] = 4.7\ \text{W}\end{aligned}$$

Therefore, the fraction of heat loss by radiation is

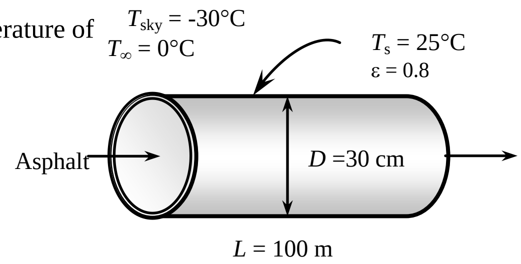
$$\text{Radiation fraction} = \frac{\dot{Q}_{radiation}}{\dot{Q}_{total}} = \frac{4.7\,\text{W}}{40\,\text{W}} = 0.1175 = 11.8\%$$

which is greater than 5%. Therefore, the radiation effect is still more than acceptable, and corrections must be made for the radiation effect.

9-31 A thick fluid flows through a pipe in calm ambient air. The pipe is heated electrically. The power rating of the electric resistance heater and the cost of electricity during a 10-h period are to be determined.√

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (25+0)/2 = 12.5°C$ are (Table A-15)



$$k = 0.02458 \text{ W/m.°C}$$
$$\upsilon = 1.448\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7330$$
$$\beta = \frac{1}{T_f} = \frac{1}{(12.5+273)\text{K}} = 0.003503 \text{K}^{-1}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003503 \text{K}^{-1})(25-0 \text{ K})(0.3 \text{ m})^3}{(1.448\times10^{-5} \text{ m}^2\text{/s})^2}(0.7330) = 8.106\times10^7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(8.106\times10^7)^{1/6}}{\left[1+\left(0.559/0.7330\right)^{9/16}\right]^{8/27}}\right\}^2 = 53.29$$

$$h = \frac{k}{L_c}Nu = \frac{0.02458 \text{ W/m.°C}}{0.3 \text{ m}}(53.29) = 4.366 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.3 \text{ m})(100 \text{ m}) = 94.25 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (4.366 \text{ W/m}^2\text{.°C})(94.25 \text{ m}^2)(25-0)\text{°C} = 10{,}287 \text{ W}$$

The radiation heat loss from the cylinder is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$= (0.8)(94.25 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(25+273\text{K})^4 - (-30+273\text{K})^4] = 18{,}808 \text{ W}$$

Then,

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural\\convection}} + \dot{Q}_{radiation} = 10{,}287 + 18{,}808 = 29{,}094 \text{ W} = \mathbf{29.1\ kW}$$

The total amount and cost of heat loss during a 10 hour period is

$$Q = \dot{Q}\Delta t = (29.1 \text{ kW})(10\text{h}) = 290.9 \text{ kWh}$$

$$\text{Cost} = (290.9 \text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$26.18}$$