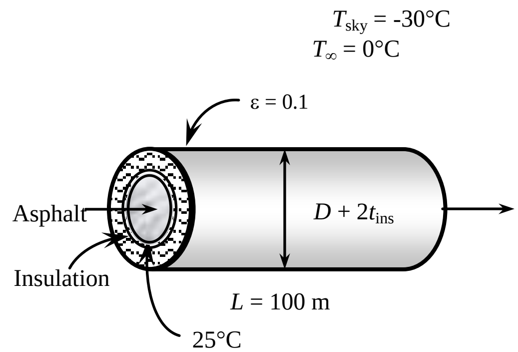
9-32 A fluid flows through a pipe in calm ambient air. The pipe is heated electrically. The thickness of the insulation needed to reduce the losses by 85% and the money saved during 10-h are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties Insulation will drop the outer surface temperature to a value close to the ambient temperature, and possible below it because of the very low sky temperature for radiation heat loss. For convenience, we use the properties of air at 1 atm and 5°C (the anticipated film temperature) (Table A-15),

$$k = 0.02401 \text{ W/m.°C}$$
$$\upsilon = 1.382\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7350$$
$$\beta = \frac{1}{T_f} = \frac{1}{(5+273)\text{K}} = 0.003597 \text{K}^{-1}$$



Analysis The rate of heat loss in the previous problem was obtained to be 29,094 W. Noting that insulation will cut down the heat losses by 85%, the rate of heat loss will be

$$\dot{Q} = (1-0.85)\dot{Q}_{\text{no insulation}} = 0.15\times 29{,}094 \text{ W} = 4364 \text{ W}$$

The amount of energy and money insulation will save during a 10-h period is simply determined from

$$Q_{saved,total} = \dot{Q}_{saved}\Delta t = (0.85\times 29.094 \text{ kW})(10\text{h}) = 247.3 \text{ kWh}$$

$$\text{Money saved} = (\text{Energy saved})(\text{Unit cost of energy}) = (247.3 \text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$22.26}$$

The characteristic length in this case is the outer diameter of the insulated pipe, $L_c = D + 2t_{insul} = 0.3 + 2t_{insul}$ where t_{insul} is the thickness of insulation in m. Then the problem can be formulated for T_s and t_{insul} as follows:

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003597 \text{K}^{-1})[(T_s - 273)\text{K}](0.3+2t_{insul})^3}{(1.382\times10^{-5} \text{ m}^2/\text{s})^2}(0.7350)$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/0.7350\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$h = \frac{k}{L_c}Nu = \frac{0.02401 \text{ W/m.°C}}{L_c}Nu$$

$$A_s = \pi D_0 L = \pi(0.3+2t_{insul})(100\text{m})$$

The total rate of heat loss from the outer surface of the insulated pipe by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)$$

$$4364 = hA_s(T_s - 273) + (0.1)A_s(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[T_s^4 - (-30+273\text{K})^4]$$

In steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

$$\dot{Q} = \dot{Q}_{insulation} = \frac{2\pi kL(T_{\text{tank}} - T_s)}{\ln(D_o/D)} \rightarrow 4364 \text{ W} = \frac{2\pi(0.035 \text{ W/m.°C})(100 \text{ m})(298 - T_s)\text{K}}{\ln[(0.3+2t_{insul})/0.3]}$$

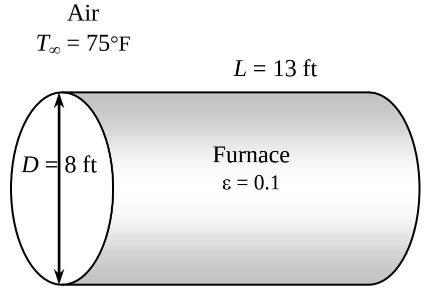
The solution of all of the equations above simultaneously using an equation solver gives T_s = 281.5 K = 8.5°C and t_{insul} = **0.013 m = 1.3 cm**.

Note that the film temperature is (8.5+0)/2 = 4.25°C which is very close to the assumed value of 5°C. Therefore, there is no need to repeat the calculations using properties at this new film temperature.

9-33E An industrial furnace that resembles a horizontal cylindrical enclosure whose end surfaces are well insulated. The highest allowable surface temperature of the furnace and the annual cost of this loss to the plant are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2=(140+75)/2=107.5$°F are (Table A-15)



$$k = 0.01546 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.1851\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7249$$
$$\beta = \frac{1}{T_f} = \frac{1}{(107.5+460)\text{R}} = 0.001762\text{R}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 140°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the furnace, $L_c = D = 8\text{ ft}$. Then,

$$Ra = \frac{g\beta(T_s-T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)(0.001762\text{R}^{-1})(140-75\text{ R})(8\text{ ft})^3}{(0.1851\times10^{-3}\text{ ft}^2\text{/s})^2}(0.7249) = 3.996\times10^{10}$$

$$Nu = \left\{0.6+\frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6+\frac{0.387(3.996\times10^{10})^{1/6}}{\left[1+(0.559/0.7249)^{9/16}\right]^{8/27}}\right\}^2 = 376.9$$

$$h = \frac{k}{D}Nu = \frac{0.01546\text{ Btu/h.ft.°F}}{8\text{ ft}}(376.9) = 0.7287\text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi DL = \pi(8\text{ ft})(13\text{ ft}) = 326.7\text{ ft}^2$$

The total rate of heat generated in the furnace is

$$\dot{Q}_{gen} = (0.82)(48\text{ therms/h})(100{,}000\text{ Btu/therm}) = 3.936\times10^6\text{ Btu/h}$$

Noting that 1% of the heat generated can be dissipated by natural convection and radiation ,

$$\dot{Q} = (0.01)(3.936\times10^6\text{ Btu/h}) = 39{,}360\text{ Btu/h}$$

The total rate of heat loss from the furnace by natural convection and radiation can be expressed as

$$\dot{Q} = hA_s(T_s-T_\infty)+\varepsilon A_s\sigma(T_s{}^4-T_{surr}{}^4)$$
$$39{,}360\text{ Btu/h} = (0.7287\text{ Btu/h.ft}^2\text{.°F})(326.7\text{ ft}^2)[T_s-(75+460\text{ R})]$$
$$+(0.85)(326.7\text{ m}^2)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\text{.R}^4)[T_s{}^4-(75+460\text{ R})^4]$$

Its solution is

$$T_s = 601.8\text{ R} = \mathbf{141.8°F}$$

which is very close to the assumed value. Therefore, there is no need to repeat calculations.

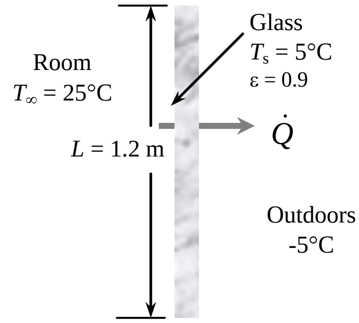
The total amount of heat loss and its cost during a-2800 hour period is

$$Q_{total} = \dot{Q}_{total}\Delta t = (39{,}360\text{ Btu/h})(2800\text{ h}) = 1.102\times10^8\text{ Btu}$$

$$\text{Cost} = (1.102\times10^8/100{,}000\text{ therm})(\$0.65/\text{therm}) = \mathbf{\$716.4}$$

9-34 A glass window is considered. The convection heat transfer coefficient on the inner side of the window, the rate of total heat transfer through the window, and the combined natural convection and radiation heat transfer coefficient on the outer surface of the window are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (5+25)/2 = 15°C$ are (Table A-15)

$$k = 0.02476 \text{ W/m.°C}$$
$$\upsilon = 1.471\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7323$$
$$\beta = \frac{1}{T_f} = \frac{1}{(15+273)\text{K}} = 0.003472 \text{K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the window, $L_c = L = 1.2$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003472 \text{ K}^{-1})(25-5 \text{ K})(1.2 \text{ m})^3}{(1.471\times10^{-5} \text{ m}^2\text{/s})^2}(0.7323) = 3.986\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.986\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7323}\right)^{9/16}\right]^{8/27}}\right\}^2 = 189.7$$

$$h = \frac{k}{L}Nu = \frac{0.02476 \text{ W/m.°C}}{1.2 \text{ m}}(189.7) = \mathbf{3.915 \text{ W/m}^2\text{.°C}}$$
$$A_s = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

(*b*) The sum of the natural convection and radiation heat transfer from the room to the window is

$$\dot{Q}_{\text{convection}} = hA_s(T_\infty - T_s) = (3.915 \text{ W/m}^2\text{.°C})(2.4 \text{ m}^2)(25-5)°\text{C} = 187.9 \text{ W}$$
$$\dot{Q}_{\text{radiation}} = \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)$$
$$= (0.9)(2.4 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(25+273 \text{K})^4 - (5+273 \text{K})^4] = 234.3 \text{ W}$$
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{radiation}} = 187.9 + 234.3 = \mathbf{422.2 \text{ W}}$$

(*c*) The outer surface temperature of the window can be determined from

$$\dot{Q}_{\text{total}} = \frac{kA_s}{t}(T_{s,i} - T_{s,o}) \longrightarrow T_{s,o} = T_{s,i} - \frac{\dot{Q}_{\text{total}}t}{kA_s} = 5°\text{C} - \frac{(346 \text{ W})(0.006 \text{ m})}{(0.78 \text{ W/m.°C})(2.4 \text{ m}^2)} = 3.65°\text{C}$$

Then the combined natural convection and radiation heat transfer coefficient on the outer window surface becomes

$$\dot{Q}_{\text{total}} = h_{\text{combined}}A_s(T_{s,o} - T_{\infty,o})$$

or
$$h_{\text{combined}} = \frac{\dot{Q}_{\text{total}}}{A_s(T_{s,o} - T_{\infty,o})} = \frac{346 \text{ W}}{(2.4 \text{ m}^2)[3.65-(-5)]°\text{C}} = \mathbf{20.35 \text{ W/m}^2\text{.°C}}$$

Note that $\Delta T = \dot{Q}R$ and thus the thermal resistance R of a layer is proportional to the temperature drop across that layer. Therefore, the fraction of thermal resistance of the glass is equal to the ratio of the temperature drop across the glass to the overall temperature difference,

$$\frac{R_{\text{glass}}}{R_{\text{total}}} = \frac{\Delta T_{\text{glass}}}{\Delta TR_{\text{total}}} = \frac{5-3.65}{25-(-5)} = 0.045 \text{ (or 4.5\%)}$$

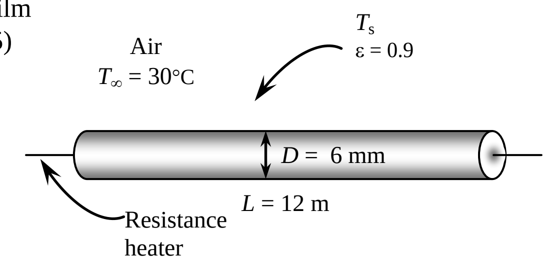
which is low. Thus it is reasonable to neglect the thermal resistance of the glass.

9-35 An insulated electric wire is exposed to calm air. The temperature at the interface of the wire and the plastic insulation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\upsilon = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 50°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the insulated wire $L_c = D = 0.006$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(0.006 \text{ m})^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 339.3$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(339.3)^{1/6}}{\left[1+\left(0.559/0.7255\right)^{9/16}\right]^{8/27}}\right\}^2 = 2.101$$

$$h = \frac{k}{D}Nu = \frac{0.02662 \text{ W/m.°C}}{0.006 \text{ m}}(2.101) = 9.327 \text{ W/m}^2\text{.°C}$$
$$A_s = \pi DL = \pi(0.006 \text{ m})(12 \text{ m}) = 0.2262 \text{ m}^2$$

The rate of heat generation, and thus the rate of heat transfer is

$$\dot{Q} = VI = (8 \text{ V})(10 \text{ A}) = 80 \text{ W}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$80 \text{ W} = (9.327 \text{ W/m}^2\text{.°C})(0.226 \text{ m}^2)(T_s - 30)\text{°C}$$
$$+ (0.9)(0.2262 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s+273)^4 - (30+273 \text{K})^4]$$

Its solution is

$$T_s = 52.6\text{°C}$$

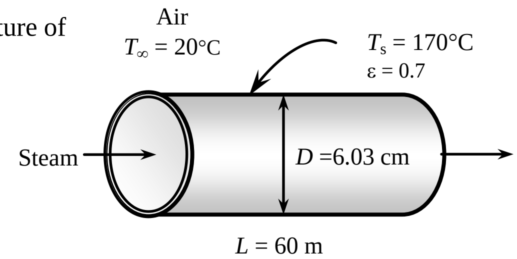
which is close to the assumed value of 50°C. Then the temperature at the interface of the wire and the plastic cover in steady operation becomes

$$\dot{Q} = \frac{2\pi kL}{\ln(D_2/D_1)}(T_i - T_s) \longrightarrow T_i = T_s + \frac{\dot{Q}\ln(D_2/D_1)}{2\pi kL} = 52.6\text{°C} + \frac{(80 \text{ W})\ln(6/3)}{2\pi(0.15 \text{ W/m.°C})(12 \text{ m})} = \mathbf{57.5°C}$$

9-36 A steam pipe extended from one end of a plant to the other with no insulation on it. The rate of heat loss from the steam pipe and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (170+20)/2 = 95°C$ are (Table A-15)



$$k = 0.0306 \text{ W/m.°C}$$
$$\upsilon = 2.252\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7121$$
$$\beta = \frac{1}{T_f} = \frac{1}{(95+273)\text{K}} = 0.002717 \text{K}^{-1}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.0603\text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002717 \text{K}^{-1})(170-20 \text{ K})(0.0603 \text{m})^3}{(2.252\times10^{-5} \text{ m}^2\text{/s})^2}(0.7121) = 1.231\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.231\times10^6)^{1/6}}{\left[1+\left(0.559/0.7121\right)^{9/16}\right]^{8/27}}\right\}^2 = 15.42$$

$$h = \frac{k}{D}Nu = \frac{0.0306 \text{ W/m.°C}}{0.0603 \text{m}}(15.42) = 7.823 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.0603 \text{m})(60 \text{m}) = 11.37 \text{ m}^2$$

Then the total rate of heat transfer by natural convection and radiation becomes

$$\begin{aligned}\dot{Q} &= hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4) \\ &= (7.823 \text{ W/m}^2\text{.°C})(11.37 \text{ m}^2)(170-20)\text{°C} \\ &\quad + (0.7)(11.37 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(170+273 \text{K})^4 - (20+273 \text{K})^4] \\ &= 27{,}388 W = \mathbf{27.4\ kW}\end{aligned}$$

The total amount of gas consumption and its cost during a one-year period is

$$Q_{gas} = \frac{\dot{Q}\Delta t}{\eta} = \frac{27.388 \text{ kJ/s}}{0.78}\left(\frac{1 \text{ therm}}{105{,}500 \text{kJ}}\right)(8760 \text{ h/yr}\times3600 \text{ s/h}) = 10{,}496 \text{ therms/yr}$$

$$\text{Cost} = (10{,}496 \text{ therms/yr})(\$0.538/\text{ therm}) = \mathbf{\$5647/yr}$$

9-37 "!PROBLEM 9-37"

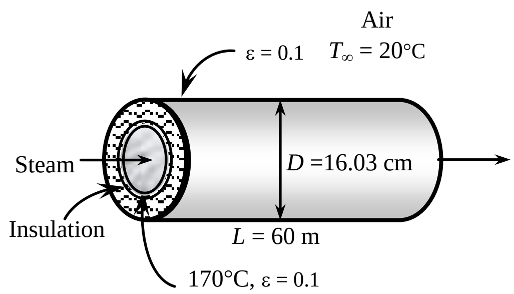
```
"GIVEN"
L=60 "[m]"
D=0.0603 "[m]"
T_s=170 "[C], parameter to be varied"
T_infinity=20 "[C]"
epsilon=0.7
T_surr=T_infinity
eta_furnace=0.78
UnitCost=0.538 "[$/therm]"
time=24*365 "[h]"

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s+T_infinity)
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"
g=9.807 "[m/s^2], gravitational acceleration"

"ANALYSIS"
delta=D
Ra=(g*beta*(T_s-T_infinity)*delta^3)/nu^2*Pr
Nusselt=(0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27))^2
h=k/delta*Nusselt
A=pi*D*L
Q_dot=h*A*(T_s-T_infinity)+epsilon*A*sigma*((T_s+273)^4-(T_surr+273)^4)
Q_gas=(Q_dot*time)/eta_furnace*Convert(h, s)*Convert(J, kJ)*Convert(kJ, therm)
Cost=Q_gas*UnitCost
```

T_s [C]	Q [W]	Cost [$]
100	11636	2399
105	12594	2597
110	13577	2799
115	14585	3007
120	15618	3220
125	16676	3438
130	17760	3661
135	18869	3890
140	20004	4124
145	21166	4364
150	22355	4609
155	23570	4859
160	24814	5116
165	26085	5378
170	27385	5646
175	28713	5920
180	30071	6200
185	31459	6486
190	32877	6778
195	34327	7077
200	35807	7382

9-38 A steam pipe extended from one end of a plant to the other. It is proposed to insulate the steam pipe for \$750. The simple payback period of the insulation to pay for itself from the energy it saves are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+20)/2 = 27.5°C$ are (Table A-15)



$$k = 0.0257 \text{ W/m.°C}$$
$$\upsilon = 1.584\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7289$$
$$\beta = \frac{1}{T_f} = \frac{1}{(27.5+273)\text{K}} = 0.003328\text{K}^{-1}$$

Analysis Insulation will drop the outer surface temperature to a value close to the ambient temperature. The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the outer surface temperature to be 35°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the insulated pipe, $L_c = D = 0.1603\text{m}.$ Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003328\text{K}^{-1})(35-20\text{ K})(0.1603\text{m})^3}{(1.584\times10^{-5}\text{ m}^2\text{/s})^2}(0.7289) = 5.856\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(5.856\times10^6)^{1/6}}{\left[1+\left(0.559/0.7289\right)^{9/16}\right]^{8/27}}\right\}^2 = 24.23$$

$$h = \frac{k}{D}Nu = \frac{0.0257 \text{ W/m.°C}}{0.1603\text{m}}(24.23) = 3.884 \text{ W/m}^2.°\text{C}$$

$$A_s = \pi DL = \pi(0.1603\text{m})(60\text{m}) = 30.22\text{ m}^2$$

Then the total rate of heat loss from the outer surface of the insulated pipe by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$= (3.884 \text{ W/m}^2.°\text{C})(30.22 \text{ m}^2)(35-20)°\text{C}$$
$$+ (0.1)(30.22 \text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(35+273\text{K})^4 - (20+273\text{K})^4]$$
$$= 2039 \text{ W}$$

In steady operation, the heat lost from the exposed surface of the insulation by convection and radiation must be equal to the heat conducted through the insulation. This requirement gives the surface temperature to be

$$\dot{Q} = \dot{Q}_{\text{insulation}} = \frac{T_{s.i} - T_s}{R_{ins}} = \frac{T_{s.i} - T_s}{\dfrac{\ln(D_2/D_1)}{2\pi kL}} \rightarrow 2039\text{ W} = \frac{(170 - T_s)°\text{C}}{\dfrac{\ln(16.03/6.03)}{2\pi(0.038\text{ W/m.°C})(60\text{ m})}}$$

It gives 30.8°C for the surface temperature, which is somewhat different than the assumed value of 35°C. Repeating the calculations with other surface temperatures gives

$$T_s = 34.3°\text{C} \quad \text{and}\ \dot{Q} = 1988\text{W}$$

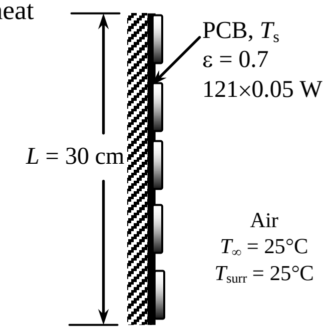
Heat loss and its cost without insulation was determined in the Prob. 9-36 to be 27.388 kW and \$5647. Then the reduction in the heat losses becomes

$$\dot{Q}_{\text{saved}} = 27.388 - 1.988 \cong 25.40\text{ kW} \quad \text{or} \quad 25.388/27.40 = 0.927 \quad (92.7\%)$$

Therefore, the money saved by insulation will be 0.921×(\$5647/yr) = **\$5237/yr** which will pay for the cost of \$750 in \$750/(\$5237/yr)=0.1432 year = **52.3 days**.

9-39 A circuit board containing square chips is mounted on a vertical wall in a room. The surface temperature of the chips is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (35+25)/2 = 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\upsilon = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7282$$
$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033 \text{K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the board, $L_c = L = 0.3 \text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{m/s}^2)(0.0033 \text{K}^{-1})(35-25 \text{K})(0.3 \text{m})^3}{(1.608\times10^{-5} \text{ m}^2\text{/s})^2}(0.7282) = 2.463\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.463\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 40.57$$

$$h = \frac{k}{L}Nu = \frac{0.02588 \text{W/m.°C}}{0.3 \text{m}}(40.57) = 3.50 \text{W/m}^2\text{.°C}$$
$$A_s = (0.3 \text{m})^2 = 0.09 \text{m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)$$
$$(121\times0.05) \text{ W} = (3.50 \text{W/m}^2\text{.°C})(0.09 \text{m}^2)(T_s - 25)\text{°C}$$
$$+ (0.7)(0.09 \text{m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s + 273\text{K})^4 - (25+273\text{K})^4]$$

Its solution is

T_s = **33.5°C**

which is sufficiently close to the assumed value in the evaluation of properties and *h*. Therefore, there is no need to repeat calculations by reevaluating the properties and *h* at the new film temperature.

9-40 A circuit board containing square chips is positioned horizontally in a room. The surface temperature of the chips is to be determined for two orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.

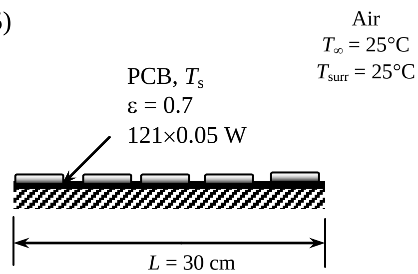
Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30°C$ are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$

$$\upsilon = 1.608\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7282$$

$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033 \text{K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. The characteristic length for both cases is determined from

$$L_c = \frac{A_s}{p} = \frac{(0.3\,\text{m})^2}{2[(0.3\,\text{m})+(0.3\,\text{m})]} = 0.075\,\text{m}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.00333\,\text{K}^{-1})(35-25\,\text{K})(0.075\,\text{m})^3}{(1.608\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7282) = 3.848\times10^5$$

(a) Chips (hot surface) facing up:

$$Nu = 0.54Ra^{1/4} = 0.54(3.848\times10^5)^{1/4} = 13.45$$

$$h = \frac{k}{L_c}Nu = \frac{0.02588\,\text{W/m.°C}}{0.075\,\text{m}}(13.45) = 4.641\,\text{W/m}^2.°\text{C}$$

$$A_s = (0.3\,\text{m})^2 = 0.09\,\text{m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$(121\times0.05)\ \text{W} = (4.641\,\text{W/m}^2.°\text{C})(0.09\,\text{m}^2)(T_s - 25)°\text{C}$$
$$+(0.7)(0.09\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(T_s + 273\text{K})^4 - (25+273\text{K})^4]$$

Its solution is T_s = **32.5°C**

which is sufficiently close to the assumed value. Therefore, there is no need to repeat calculations.

(b) Chips (hot surface) facing up:

$$Nu = 0.27Ra^{1/4} = 0.27(3.848\times10^5)^{1/4} = 6.725$$

$$h = \frac{k}{L_c}Nu = \frac{0.02588\,\text{W/m.°C}}{0.075\,\text{m}}(6.725) = 2.321\,\text{W/m}^2.°\text{C}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

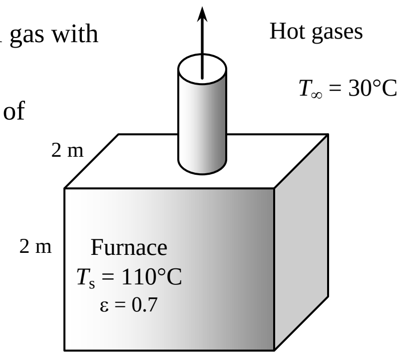
$$(121\times0.05)\ \text{W} = (2.321\,\text{W/m}^2.°\text{C})(0.09\,\text{m}^2)(T_s - 25)°\text{C}$$
$$+(0.7)(0.09\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(T_s + 273\text{K})^4 - (25+273\text{K})^4]$$

Its solution is T_s = **35.0°C**

which is identical to the assumed value in the evaluation of properties and h. Therefore, there is no need to repeat calculations.

9-41 It is proposed that the side surfaces of a cubic industrial furnace be insulated for $550 in order to reduce the heat loss by 90 percent. The thickness of the insulation and the payback period of the insulation to pay for itself from the energy it saves are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (110+30)/2 = 70$°C are (Table A-15)

$$k = 0.02881 \text{ W/m.°C}$$
$$\upsilon = 1.995\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the height of the furnace, $L_c = L = 2$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.002915\text{ K}^{-1})(110-30\text{ K})(2\text{ m})^3}{(1.995\times10^{-5}\text{ m}^2\text{/s})^2}(0.7177) = 3.301\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.301\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7177}\right)^{9/16}\right]^{8/27}}\right\}^2 = 369.2$$

$$h = \frac{k}{L_c}Nu = \frac{0.02881\text{ W/m.°C}}{2\text{ m}}(369.2) = 5.318\text{ W/m}^2\text{.°C}$$

$$A_s = 4(2\text{ m})^2 = 16\text{ m}^2$$

Then the heat loss by combined natural convection and radiation becomes

$$\begin{aligned}\dot{Q} &= hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) \\ &= (5.318\text{ W/m}^2\text{.°C})(16\text{ m}^2)(110-30)\text{°C} \\ &\quad + (0.7)(16\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(110+273\text{K})^4 - (30+273\text{K})^4] \\ &= 15{,}119\text{ W}\end{aligned}$$

Noting that insulation will reduce the heat losses by 90%, the rate of heat loss after insulation will be

$$\dot{Q}_{\text{saved}} = 0.9\dot{Q}_{\text{no insulation}} = 0.9\times15{,}119\text{ W} = 13{,}607\text{ W}$$
$$\dot{Q}_{\text{loss}} = (1-0.9)\dot{Q}_{\text{no insulation}} = 0.1\times15{,}119\text{ W} = 1512\text{ W}$$

The furnace operates continuously and thus 8760 h. Then the amount of energy and money the insulation will save becomes

$$\text{Energy saved} = \dot{Q}_{saved}\Delta t = \frac{13.607\text{ kJ/s}}{0.78}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right)(8760\times3600\text{ s/yr}) = 5215\text{ therms/yr}$$

$$\text{Money saved} = (\text{Energy saved})(\text{Unit cost of energy}) = (5215\text{ therms})(\$0.55/\text{therm}) = \$2868$$

Therefore, the money saved by insulation will pay for the cost of $550 in

550/($2868/yr)=0.1918 yr = **70 days**.

Insulation will lower the outer surface temperature, the Rayleigh and Nusselt numbers, and thus the convection heat transfer coefficient. For the evaluation of the heat transfer coefficient, we assume the surface temperature in this case to be 50°C. The properties of air at the film temperature of $(T_s+T_\infty)/2$ = (50+30)/2 = 40°C are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\upsilon = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{K}^{-1}$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{K}^{-1})(50-30 \text{ K})(2 \text{ m})^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 1.256\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.256\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2 = 272.0$$

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{2 \text{ m}}(272.0) = 3.620 \text{ W/m}^2\text{.°C}$$

$$A_s = 4\times(2 \text{ m})(2+2t_{insul}) \text{ m}$$

The total rate of heat loss from the outer surface of the insulated furnace by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$1512 \text{ W} = (3.620 \text{ W/m}^2\text{.°C})A(T_s - 30)\text{°C} + (0.7)A(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s + 273\text{K})^4 - (30+273\text{K})^4]$$

I

n steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

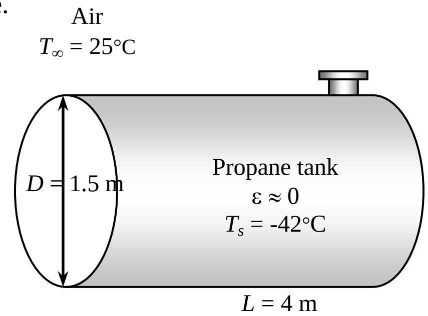
$$\dot{Q} = \dot{Q}_{insulation} = kA_s\frac{(T_{\text{furnace}} - T_s)}{t_{ins}} \rightarrow 1512 \text{ W} = (0.038 \text{ W/m.°C})A_s\frac{(110 - T_s)\text{°C}}{t_{insul}}$$

Solving the two equations above by trial-and error (or better yet, an equation solver) gives

T_s = 48.4°C and t_{insul} = 0.0254 m = **2.54 cm**

9-42 A cylindrical propane tank is exposed to calm ambient air. The propane is slowly vaporized due to a crack developed at the top of the tank. The time it will take for the tank to empty is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation heat transfer is negligible.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (-42+25)/2 = -8.5°C are (Table A-15)

$$k = 0.02299 \text{ W/m.°C}$$
$$\upsilon = 1.265\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7383$$
$$\beta = \frac{1}{T_f} = \frac{1}{(-8.5+273)\text{K}} = 0.003781 \text{ K}^{-1}$$

Analysis The tank gains heat through its cylindrical surface as well as its circular end surfaces. For convenience, we take the heat transfer coefficient at the end surfaces of the tank to be the same as that of its side surface. (The alternative is to treat the end surfaces as a vertical plate, but this will double the amount of calculations without providing much improvement in accuracy since the area of the end surfaces is much smaller and it is circular in shape rather than being rectangular). The characteristic length in this case is the outer diameter of the tank, $L_c = D = 1.5$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003781 \text{ K}^{-1})[(25-(-42)\text{ K}](1.5 \text{ m})^3}{(1.265\times10^{-5} \text{ m}^2\text{/s})^2}(0.7383) = 3.869\times10^{10}$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(3.869\times10^{10})^{1/6}}{\left[1+\left(0.559/0.7383\right)^{9/16}\right]^{8/27}}\right\}^2 = 374.1$$

$$h = \frac{k}{D}Nu = \frac{0.02299 \text{ W/m.°C}}{1.5 \text{ m}}(374.1) = 5.733 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL + 2\pi D^2/4 = \pi(1.5 \text{ m})(4 \text{ m}) + 2\pi(1.5 \text{ m})^2/4 = 22.38 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (5.733 \text{ W/m}^2\text{.°C})(22.38 \text{ m}^2)[(25-(-42)]\text{°C} = 8598 \text{ W}$$

The total mass and the rate of evaporation of propane are

$$m = \rho V = \rho\frac{\pi D^2}{4}L = (581 \text{ kg/m}^3)\frac{\pi(1.5 \text{ m})^2}{4}(4 \text{ m}) = 4107 \text{ kg}$$

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{8.598 \text{ kJ/s}}{425 \text{ kJ/kg}} = 0.02023 \text{ kg/s}$$

and it will take

$$\Delta t = \frac{m}{\dot{m}} = \frac{4107 \text{ kg}}{0.02023 \text{ kg/s}} = 202{,}996 \text{ s} = \mathbf{56.4 \text{ hours}}$$

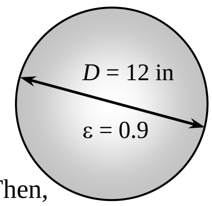
for the propane tank to empty.

9-43E The average surface temperature of a human head is to be determined when it is not covered.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The head can be approximated as a 12-in.-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 120°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (120+77)/2 = 98.5$°F are (Table A-15E)

$$k = 0.01525 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.180\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7262$$
$$\beta = \frac{1}{T_f} = \frac{1}{(98.5+460)\text{R}} = 0.001791\text{R}^{-1}$$



Analysis The characteristic length for a spherical object is $L_c = D = 12/24 = 0.5$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001791\text{R}^{-1})(95-77\text{ R})(0.5\text{ ft})^3}{(0.180\times10^{-3}\text{ ft}^2\text{/s})^2}(0.7262) = 6.943\times10^6$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(6.943\times10^6)^{1/4}}{\left[1+\left(\frac{0.469}{0.7262}\right)^{9/16}\right]^{4/9}} = 25.39$$

$$h = \frac{k}{D}Nu = \frac{0.01525 \text{ Btu/h.ft.°F}}{1\text{ ft}}(25.39) = 0.7744 \text{ Btu/h.ft}^2\text{.°F}$$
$$A_s = \pi D^2 = \pi(0.5\text{ ft})^2 = 0.7854\text{ ft}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$(287/4 \text{ Btu/h}) = (0.7744 \text{ Btu/h.ft}^2\text{.°F})(0.7854\text{ ft}^2)(T_s - 77)\text{°F}$$
$$+(0.9)(0.7854\text{ m}^2)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\text{.R}^4)[(T_s+460\text{ R})^4 - (77+460\text{ R})^4]$$

Its solution is

T_s = **125.9°F**

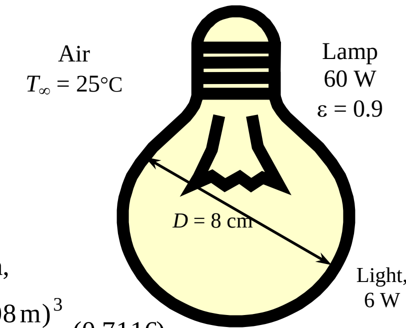
which is sufficiently close to the assumed value in the evaluation of the properties and h. Therefore, there is no need to repeat calculations.

9-44 The equilibrium temperature of a light glass bulb in a room is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The light bulb is approximated as an 8-cm-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 170°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (170+25)/2 = 97.5°C are (Table A-15)

$$k = 0.03077 \text{ W/m.°C}$$
$$\upsilon = 2.279\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7116$$
$$\beta = \frac{1}{T_f} = \frac{1}{(97.5+273)\text{K}} = 0.002699\text{K}^{-1}$$



Analysis The characteristic length in this case is $L_c = D$ = 0.08 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.002699\text{K}^{-1})(170-25\text{ K})(0.08\text{ m})^3}{(2.279\times10^{-5}\text{ m}^2/\text{s})^2}(0.7116)$$
$$= 2.694\times10^6$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\Pr)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(2.694\times10^6)^{1/4}}{\left[1+(0.469/0.7116)^{9/16}\right]^{4/9}} = 20.42$$

Then

$$h = \frac{k}{D}Nu = \frac{0.03077\text{ W/m.°C}}{0.08\text{ m}}(20.42) = 7.854\text{ W/m}^2.\text{°C}$$
$$A_s = \pi D^2 = \pi(0.08\text{ m})^2 = 0.02011\text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$(0.90\times60)\text{ W} = (7.854\text{ W/m}^2.\text{°C})(0.02011\text{ m}^2)(T_s - 25)\text{°C}$$
$$+(0.9)(0.02011\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(T_s+273)^4 - (25+273\text{K})^4]$$

Its solution is

T_s = **169.4°C**

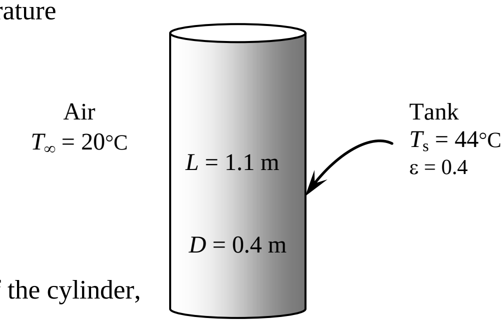
which is sufficiently close to the value assumed in the evaluation of properties and *h*. Therefore, there is no need to repeat calculations.

9-45 A vertically oriented cylindrical hot water tank is located in a bathroom. The rate of heat loss from the tank by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the tank is constant.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (44+20)/2 = 32°C$ are (Table A-15)

$$k = 0.02603 \text{ W/m.°C}$$
$$\upsilon = 1.627\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7276$$
$$\beta = \frac{1}{T_f} = \frac{1}{(32+273)\text{K}} = 0.003279 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the cylinder, $L_c = L = 1.1 \text{ m}$. Then,

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2} = \frac{(9.81 \text{ m/s}^2)(0.003279 \text{ K}^{-1})(44-20 \text{ K})(1.1 \text{ m})^3}{(1.627\times10^{-5} \text{ m}^2\text{/s})^2} = 3.883\times10^9$$

A vertical cylinder can be treated as a vertical plate when

$$D(= 0.4 \text{ m}) \geq \frac{35L}{\text{Gr}^{1/4}} = \frac{35(1.1 \text{ m})}{(3.883\times10^9)^{1/4}} = 0.1542 \text{ m}$$

which is satisfied. That is, the Nusselt number relation for a vertical plate can be used for the side surfaces. For the top and bottom surfaces we use the relevant Nusselt number relations. First, for the side surfaces,

$$\text{Ra} = \text{GrPr} = (3.883\times10^9)(0.7276) = 2.825\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.825\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7276}\right)^{9/16}\right]^{8/27}}\right\}^2 = 170.2$$

$$h = \frac{k}{L}Nu = \frac{0.02603 \text{ W/m.°C}}{1.1 \text{ m}}(170.2) = 4.027 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.4 \text{ m})(1.1 \text{ m}) = 1.382 \text{ m}^2$$

$$\dot{Q}_{\text{side}} = hA_s(T_s - T_\infty) = (4.027 \text{ W/m}^2\text{.°C})(1.382 \text{ m}^2)(44-20)\text{°C} = 133.6 \text{ W}$$

For the top surface,

$$L_c = \frac{A_s}{p} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = \frac{0.4 \text{ m}}{4} = 0.1 \text{ m}$$

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003279 \text{ K}^{-1})(44-20 \text{ K})(0.1 \text{ m})^3}{(1.627\times10^{-5} \text{ m}^2\text{/s})^2}(0.7276) = 2.123\times10^6$$

$$\text{Nu} = 0.54\text{Ra}^{1/4} = 0.54(2.123\times10^6)^{1/4} = 20.61$$

$$h = \frac{k}{L_c}Nu = \frac{0.02603 \text{ W/m.°C}}{0.1 \text{ m}}(20.61) = 5.365 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D^2/4 = \pi(0.4 \text{ m})^2/4 = 0.1257 \text{ m}^2$$

$$\dot{Q}_{\text{top}} = hA_s(T_s - T_\infty) = (5.365 \text{ W/m}^2\text{.°C})(0.1257 \text{ m}^2)(44-20)\text{°C} = 16.2 \text{ W}$$

For the bottom surface,

$$\text{Nu} = 0.27\text{Ra}^{1/4} = 0.27(2.123\times10^6)^{1/4} = 10.31$$

$$h = \frac{k}{L_c} Nu = \frac{0.02603 \text{ W/m.°C}}{0.1 \text{ m}}(10.31) = 2.683 \text{ W/m}^2.\text{°C}$$

$$\dot{Q}_{\text{bottom}} = hA_s(T_s - T_\infty) = (2.683 \text{ W/m}^2.\text{°C})(0.1257 \text{ m}^2)(44-20)\text{°C} = 8.1 \text{ W}$$

The total heat loss by natural convection is

$$\dot{Q}_{\text{conv}} = \dot{Q}_{\text{side}} + \dot{Q}_{\text{top}} + \dot{Q}_{\text{bottom}} = 133.6 + 16.2 + 8.1 = \mathbf{157.9\ W}$$

The radiation heat loss from the tank is

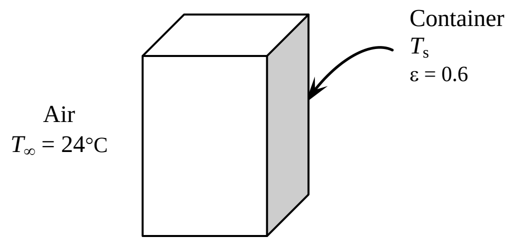
$$\begin{aligned} \dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma(T_s{}^4 - T_{surr}{}^4) \\ &= (0.4)(1.382 + 0.1257 + 0.1257 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(44+273\text{K})^4 - (20+273\text{K})^4\right] \\ &= \mathbf{101.1\ W} \end{aligned}$$

9-46 A rectangular container filled with cold water is gaining heat from its surroundings by natural convection and radiation. The water temperature in the container after a 3 hours and the average rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer coefficient at the top and bottom surfaces is the same as that on the side surfaces.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (10+24)/2 = 17°C are (Table A-15)

$$k = 0.02491 \text{ W/m.°C}$$
$$\upsilon = 1.489\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7317$$
$$\beta = \frac{1}{T_f} = \frac{1}{(17+273)\text{K}} = 0.003448 \text{K}^{-1}$$



The properties of water at 2°C are (Table A-7)

$$\rho = 1000 \text{ kg/m}^3 \text{ and } C_p = 4214 \text{ J/kg.°C}$$

Analysis We first evaluate the heat transfer coefficient on the side surfaces. The characteristic length in this case is the height of the container,

$L_c = L = 0.28$ m. Then,

$$\text{Ra} = \frac{g\beta(T_\infty - T_s)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003448 \text{K}^{-1})(24-10 \text{ K})(0.28 \text{ m})^3}{(1.489\times10^{-5} \text{ m}^2\text{/s})^2}(0.7317) = 1.133\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.133\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7317}\right)^{9/16}\right]^{8/27}}\right\}^2 = 30.52$$

$$h = \frac{k}{L}Nu = \frac{0.02491 \text{ W/m.°C}}{0.28 \text{ m}}(30.52) = 4.224 \text{ W/m}^2\text{.°C}$$

$$A_s = 2(0.28\times0.18 + 0.28\times0.18 + 0.18\times0.18) = 0.2664 \text{m}^2$$

The rate of heat transfer can be expressed as

$$\dot{Q} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s\left(T_\infty - \frac{T_1+T_2}{2}\right) + \varepsilon\sigma A_s\left[T_{\text{surr}}^4 - \left(\frac{T_1+T_2}{2}\right)^4\right]$$

$$= (4.224 \text{ W/m}^2\text{.°C})(0.2664 \text{m}^2)\left[297 - \left(\frac{275+T_2}{2}\right)\right] \quad \text{(Eq. 1)}$$

$$+ (0.6)(0.2664 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[297^4 - \left(\frac{275+T_2}{2}\right)^4\right]$$

where $(T_1+T_2)/2$ is the average temperature of water (or the container surface). The mass of water in the container is

$$m = \rho V = (1000 \text{ kg/m}^3)(0.28\times0.18\times0.18)\text{m}^3 = 9.072 \text{ kg}$$

Then the amount of heat transfer to the water is

$$Q = mC_p(T_2 - T_1) = (9.072 \text{ kg})(4214 \text{ J/kg.°C})(T_2 - 275)\text{°C} = 38{,}229(T_2 - 275)$$

The average rate of heat transfer can be expressed as

$$\dot{Q} = \frac{Q}{\Delta t} = \frac{38{,}229(T_2 - 275)}{3\times3600 \text{ s}} = 3.53976(T_2 - 275) \quad \text{(Eq. 2)}$$

Setting Eq. 1 and Eq. 2 equal to each other, we obtain the final water temperature.

$$T_2 = 284.7\ \text{K} = \mathbf{11.7°C}$$

We could repeat the solution using air properties at the new film temperature using this value to increase the accuracy. However, this would only affect the heat transfer value somewhat, which would not have significant effect on the final water temperature. The average rate of heat transfer can be determined from Eq. 2

$$\dot{Q} = 3.53976(11.7 - 2) = \mathbf{34.3\ W}$$

9-47 "!PROBLEM 9-47"

```
"GIVEN"
height=0.28 "[m]"
L=0.18 "[m]"
w=0.18 "[m]"
T_infinity=24 "[C]"
T_w1=2 "[C]"
epsilon=0.6
T_surr=T_infinity
"time=3 [h], parameter to be varied"

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_w_ave+T_infinity)
T_w_ave=1/2*(T_w1+T_w2)
rho_w=Density(water, T=T_w_ave, P=101.3)
C_p_w=CP(water, T=T_w_ave, P=101.3)*Convert(kJ/kg-C, J/kg-C)
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"
g=9.807 "[m/s^2], gravitational acceleration"

"ANALYSIS"
delta=height
Ra=(g*beta*(T_infinity-T_w_ave)*delta^3)/nu^2*Pr
Nusselt=0.59*Ra^0.25
h=k/delta*Nusselt
A=2*(height*L+height*w+w*L)
Q_dot=h*A*(T_infinity-T_w_ave)+epsilon*A*sigma*((T_surr+273)^4-(T_w_ave+273)^4)

m_w=rho_w*V_w
V_w=height*L*w
Q=m_w*C_p_w*(T_w2-T_w1)
Q_dot=Q/(time*Convert(h, s))
```

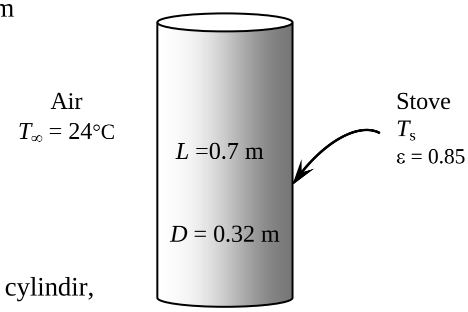
time [h]	**T_{w2} [C]**
0.5	4.013
1	5.837
1.5	7.496
2	9.013
2.5	10.41
3	11.69
3.5	12.88
4	13.98
4.5	15
5	15.96
5.5	16.85
6	17.69
6.5	18.48
7	19.22
7.5	19.92
8	20.59
8.5	21.21
9	21.81
9.5	22.37
10	22.91

9-48 A room is to be heated by a cylindrical coal-burning stove. The surface temperature of the stove and the amount of coal burned during a 30-day-period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the stove is constant. **5** The heat transfer from the bottom surface is negligible. **6** The heat transfer coefficient at the top surface is the same as that on the side surface.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (130+24)/2 = 77°C$ are (Table A-1)

$$k = 0.02931 \text{ W/m.°C}$$
$$\upsilon = 2.066\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7161$$
$$\beta = \frac{1}{T_f} = \frac{1}{(77+273)\text{K}} = 0.002857 \text{K}^{-1}$$



Analysis The characteristic length in this case is the height of the cylindir, $L_c = L = 0.7$ m. Then,

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2} = \frac{(9.81 \text{ m/s}^2)(0.002857 \text{K}^{-1})(130-24 \text{ K})(0.70 \text{ m})^3}{(2.066\times10^{-5} \text{ m}^2\text{/s})^2} = 2.387\times10^9$$

A vertical cylinder can be treated as a vertical plate when

$$D(=0.32 \text{ m}) \geq \frac{35L}{\text{Gr}^{1/4}} = \frac{35(0.7 \text{ m})}{(2.387\times10^9)^{1/4}} = 0.1108 \text{m}$$

which is satisfied. That is, the Nusselt number relation for a vertical plate can be used for side surfaces.

$$\text{Ra} = \text{GrPr} = (2.387\times10^9)(0.7161) = 1.709\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.709\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7161}\right)^{9/16}\right]^{8/27}}\right\}^2 = 145.2$$

$$h = \frac{k}{L}Nu = \frac{0.02931 \text{ W/m.°C}}{0.7 \text{ m}}(145.2) = 6.080 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL + \pi D^2/4 = \pi(0.32 \text{ m})(0.7 \text{ m}) + \pi(0.32 \text{ m})^2/4 = 0.7841 \text{m}^2$$

Then the surface temperature of the stove is determined from

$$\dot{Q} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s{}^4 - T_{\text{surr}}{}^4)$$

$$1200 \text{ W} = (6.080 \text{ W/m}^2\text{.°C})(0.7841 \text{m}^2)(T_s - 297) + (0.85)(0.7841 \text{m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)(T_s{}^4 - 290^4)$$

$$\longrightarrow T_s = 400.6 \text{ K} = \mathbf{127.6°C}$$

The amount of coal used is determined from

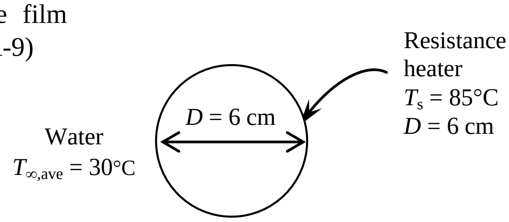
$$Q = \dot{Q}\Delta t = (1.2 \text{ kJ/s})(14 \text{ h/day}\times3600 \text{s/h}) = 60{,}480 \text{kJ}$$

$$m_{\text{coal}} = \frac{Q/\eta}{HV} = \frac{(60{,}480 \text{kJ})/0.65}{30{,}000 \text{kJ/kg}} = \mathbf{3.102 kg}$$

9-49 Water in a tank is to be heated by a spherical heater. The heating time is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature of the outer surface of the sphere is constant.

Properties Using the average temperature for water (15+45)/2=30 as the fluid temperature, the properties of water at the film temperature of $(T_s+T_\infty)/2$ = (85+30)/2 = 57.5°C are (Table A-9)



$$k = 0.6515 \text{ W/m.°C}$$
$$\upsilon = 0.474\times10^{-6} \text{ m}^2\text{/s}$$
$$\text{Pr} = 3.12$$
$$\beta = 0.501\times10^{-3} \text{ K}^{-1}$$

Also, the properties of water at 30°C are (Table A-9)

$$\rho = 996 \text{ kg/m}^3 \text{ and } C_p = 4178 \text{ J/kg.°C}$$

Analysis The characteristic length in this case is $L_c = D$ = 0.06 m. Then,

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.501\times10^{-3} \text{ K}^{-1})(85-30 \text{ K})(0.06 \text{ m})^3}{(0.474\times10^{-6} \text{ m}^2\text{/s})^2}(3.12) = 8.108\times10^8$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(0.469/\text{Pr}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(8.108\times10^8)^{1/4}}{\left[1+\left(0.469/3.12\right)^{9/16}\right]^{4/9}} = 89.14$$

$$h = \frac{k}{D}Nu = \frac{0.6515 \text{ W/m.°C}}{0.06 \text{ m}}(89.14) = 967.9 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D^2 = \pi(0.06 \text{ m})^2 = 0.01131 \text{m}^2$$

The rate of heat transfer by convection is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (967.9 \text{ W/m}^2\text{.°C})(0.01131 \text{m}^2)(85-30) = 602.1 \text{ W}$$

The mass of water in the container is

$$m = \rho V = (996 \text{ kg/m}^3)(0.040 \text{m}^3) = 39.84 \text{ kg}$$

The amount of heat transfer to the water is

$$Q = mC_p(T_2 - T_1) = (39.84 \text{ kg})(4178 \text{ J/kg.°C})(45\text{-}15)\text{°C} = 4.994\times10^6 \text{ J}$$

Then the time the heater should be on becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{4.994\times10^6 \text{ J}}{602.1 \text{J/s}} = 8294 \text{s} = \mathbf{2.304 hours}$$