Review Problems

9-94E A small cylindrical resistor mounted on the lower part of a vertical circuit board. The approximate surface temperature of the resistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** Heat transfer through the connecting wires is negligible.

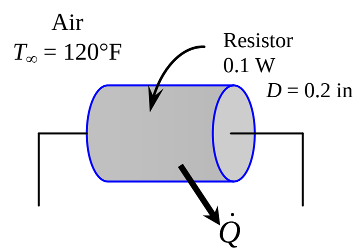
Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (220+120)/2 = 170°F are (Table A-15E)

$$k = 0.01692\ \text{Btu/h.ft.°F}$$

$$\upsilon = 0.222\times10^{-3}\ \text{ft}^2/\text{s}$$

$$\text{Pr} = 0.7161$$

$$\beta = \frac{1}{T_f} = \frac{1}{(170+460)\text{R}} = 0.001587\text{R}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 220°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the diameter of resistor, $L_c = D = 0.2$ in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(32.2\ \text{ft/s}^2)(0.001587\text{R}^{-1})(220-120\ \text{R})(0.2/12\ \text{ft})^3}{(0.222\times10^{-3}\ \text{ft}^2/\text{s})^2}(0.7161) = 343.8$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(343.8)^{1/6}}{\left[1+(0.559/0.7161)^{9/16}\right]^{8/27}}\right\}^2 = 2.105$$

$$h = \frac{k}{D}Nu = \frac{0.01692\ \text{Btu/h.ft.°F}}{0.2/12\ \text{ft}}(2.105) = 2.138\ \text{Btu/h.ft}^2.\text{°F}$$

$$A_s = \pi DL + 2D^2/4 = \pi(0.2/12\ \text{ft})(0.3/12\ \text{ft}) + 2\pi(0.2/12\ \text{ft})^2/4 = 0.00175\ \text{ft}^2$$

and $$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 120\text{°F} + \frac{(0.1\times3.412)\ \text{Btu/h}}{(2.138\ \text{Btu/h.ft}^2.\text{°F})(0.00175\ \text{ft}^2)} = \mathbf{211.5°F}$$

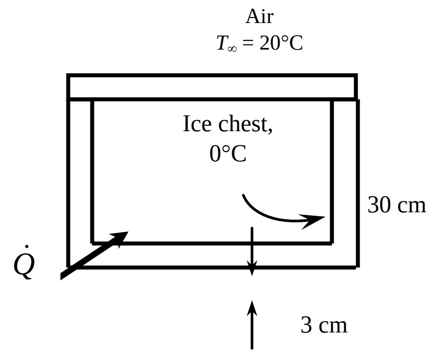
which is sufficiently close to the assumed temperature for the evaluation of properties. Therefore, there is no need to repeat calculations.

9-95 An ice chest filled with ice at 0°C is exposed to ambient air. The time it will take for the ice in the chest to melt completely is to be determined for natural and forced convection cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the base of the ice chest is disregarded. **4** Radiation effects are negligible. **5** Heat transfer coefficient is the same for all surfaces considered. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (15+20)/2 = 17.5°C are (Table A-15)

$$k = 0.02495 \text{ W/m.°C}$$
$$\upsilon = 1.493\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7316$$
$$\beta = \frac{1}{T_f} = \frac{1}{(17.5+273)\text{K}} = 0.003442 \text{ K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 15°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length for the side surfaces is the height of the chest, $L_c = L$ = 0.3 m Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003442 \text{ K}^{-1})(20-15 \text{ K})(0.3 \text{ m})^3}{(1.493\times10^{-5} \text{ m}^2/\text{s})^2}(0.7316) = 1.495\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.495\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7316}\right)^{9/16}\right]^{8/27}}\right\}^2 = 35.15$$

$$h = \frac{k}{L}Nu = \frac{0.02495 \text{ W/m.°C}}{0.3 \text{ m}}(35.15) = 2.923 \text{ W/m}^2.°\text{C}$$

The heat transfer coefficient at the top surface can be determined similarly. However, the top surface constitutes only about one-fourth of the heat transfer area, and thus we can use the heat transfer coefficient for the side surfaces for the top surface also for simplicity. The heat transfer surface area is

$$A_s = 4(0.3 \text{ m})(0.4 \text{ m}) + (0.4 \text{ m})(0.4 \text{ m}) = 0.64 \text{ m}^2$$

Then the rate of heat transfer becomes

$$\dot{Q} = \frac{T_\infty - T_{s,i}}{R_{wall} + R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\frac{L}{kA_s} + \frac{1}{hA_s}} = \frac{(20-0)°\text{C}}{\frac{0.03 \text{ m}}{(0.033 \text{ W/m.°C})(0.64 \text{ m}^2)} + \frac{1}{(2.923 \text{ W/m}^2.°\text{C})(0.64 \text{ m}^2)}} = 10.23 \text{ W}$$

The outer surface temperature of the ice chest is determined from Newton's law of cooling to be

$$\dot{Q} = hA_s(T_\infty - T_s) \rightarrow T_s = T_\infty - \frac{\dot{Q}}{hA_s} = 20°\text{C} - \frac{10.4 \text{ W}}{(2.923 \text{ W/m}^2.\text{C})(0.64 \text{ m}^2)} = 14.53°\text{C}$$

which is almost identical to the assumed value of 15°C used in the evaluation of properties and *h*. Therefore, there is no need to repeat the calculations. Then the rate at which the ice will melt becomes

$$\dot{Q} = \dot{m}h_{if} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{10.23\times10^{-3} \text{ kJ/s}}{333.7 \text{ kJ/kg}} = 3.066\times10^{-5} \text{ kg/s}$$

Therefore, the melting of the ice in the chest completely will take

$$m = \dot{m}\Delta t \longrightarrow \Delta t = \frac{m}{\dot{m}} = \frac{30 \text{ kg}}{3.066\times10^{-5} \text{ kg/s}} = 9.786\times10^{5} \text{ s} = \mathbf{271.8 h} = \mathbf{11.3 days}$$

(b) The temperature drop across the styrofoam will be much greater in this case than that across thermal boundary layer on the surface. Thus we assume outer surface temperature of the styrofoam to be $19\,°\text{C}$. Radiation heat transfer will be neglected. The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (19+20)/2 = 19.5°C are (Table A-15)

$$k = 0.0251 \text{W/m.°C}$$

$$\upsilon = 1.512\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7311$$

$$\beta = \frac{1}{T_f} = \frac{1}{(19.5+273)\text{K}} = 0.00342 \text{K}^{-1}$$

The characteristic length in this case is the width of the chest, $L_c = W$ =0.4 m. Then,

$$\text{Re} = \frac{V_\infty W}{\upsilon} = \frac{(50\times1000/3600 \text{ m/s})(0.4 \text{ m})}{1.512\times10^{-5} \text{ m}^2\text{/s}} = 367{,}538$$

which is less than critical Reynolds number (5×10^5). Therefore the flow is laminar, and the Nusselt number is determined from

$$Nu = \frac{hW}{k} = 0.664 \text{Re}^{0.5} \text{Pr}^{1/3} = 0.664(367{,}538)^{0.5}(0.7311)^{1/3} = 362.6$$

$$h = \frac{k}{W} Nu = \frac{0.0251 \text{W/m.°C}}{0.4 \text{ m}}(362.6) = 22.76 \text{ W/m}^2\text{.°C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \frac{T_\infty - T_{s,i}}{R_{wall} + R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\frac{L}{kA_s} + \frac{1}{hA_s}} = \frac{(20-0)\text{°C}}{\frac{0.03 \text{m}}{(0.033 \text{W/m.°C})(0.64 \text{ m}^2)} + \frac{1}{(22.76 \text{ W/m}^2\text{.°C})(0.64 \text{ m}^2)}} = 13.43 \text{W}$$

The outer surface temperature of the ice chest is determined from Newton’s law of cooling to be

$$\dot{Q} = hA_s(T_\infty - T_s) \rightarrow T_s = T_\infty - \frac{\dot{Q}}{hA_s} = 20\text{°C} - \frac{13.43 \text{W}}{(22.76 \text{W/m}^2\text{.C})(0.64 \text{ m}^2)} = 19.1\text{°C}$$

which is almost identical to the assumed value of 19°C used in the evaluation of properties and h. Therefore, there is no need to repeat the calculations. Then the rate at which the ice will melt becomes

$$\dot{Q} = \dot{m}h_{if} \rightarrow \dot{m} = \frac{\dot{Q}}{h_{if}} = \frac{13.43\times10^{-3} \text{ kJ/s}}{333.7 \text{ kJ/kg}} = 4.025\times10^{-5} \text{ kg/s}$$

Therefore, the melting of the ice in the chest completely will take

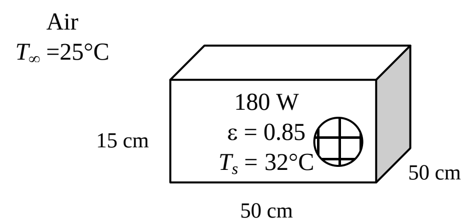
$$m = \dot{m}\Delta t \longrightarrow \Delta t = \frac{m}{\dot{m}} = \frac{30}{4.025\times10^{-5}} = 7.454\times10^{5} \text{ s} = \mathbf{207.05 h} = \mathbf{8.6 days}$$

9-96 An electronic box is cooled internally by a fan blowing air into the enclosure. The fraction of the heat lost from the outer surfaces of the electronic box is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the base surface is disregarded. **4** The pressure of air inside the enclosure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of
$(T_s+T_\infty)/2$ = (32+15)/2 = 28.5°C are (Table A-15)

$$k = 0.02577\ \text{W/m.°C}$$
$$\upsilon = 1.594\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7286$$
$$\beta = \frac{1}{T_f} = \frac{1}{(28.5+273)\text{K}} = 0.003317\ \text{K}^{-1}$$



Analysis Heat loss from the horizontal top surface:

The characteristic length in this case is $L_c = \dfrac{A_s}{p} = \dfrac{(0.5\ \text{m})^2}{2[(0.5\ \text{m})+(0.5\ \text{m})]} = 0.125\ \text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003317\ \text{K}^{-1})(32-25\ \text{K})(0.125\ \text{m})^3}{(1.594\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7286) = 1.275\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.275\times10^6)^{1/4} = 18.15$$

$$h = \frac{k}{L_c}Nu = \frac{0.02577\ \text{W/m.°C}}{0.125\ \text{m}}(18.15) = 3.741\ \text{W/m}^2.\text{°C}$$

$$A_{top} = (0.5\ \text{m})^2 = 0.25\ \text{m}^2$$

and $\qquad \dot{Q}_{top} = hA_{top}(T_s - T_\infty) = (3.741\ \text{W/m}^2.\text{°C})(0.25\ \text{m}^2)(32-25)\text{°C} = 6.55\ \text{W}$

Heat loss from vertical side surfaces:

The characteristic length in this case is the height of the box $L_c = L$ =0.15 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003317\ \text{K}^{-1})(32-25\ \text{K})(0.15\ \text{m})^3}{(1.594\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7286) = 2.204\times10^6$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.204\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7286}\right)^{9/16}\right]^{8/27}}\right\}^2 = 20.55$$

$$h = \frac{k}{L}Nu = \frac{0.02577\ \text{W/m.°C}}{0.15\ \text{m}}(20.55) = 3.530\ \text{W/m}^2.\text{°C}$$

$$A_{side} = 4(0.15\ \text{m})(0.5\ \text{m}) = 0.3\ \text{m}^2$$

and

$$\dot{Q}_{side} = hA_{side}(T_s - T_\infty) = (3.530\ \text{W/m}^2.\text{°C})(0.3\ \text{m}^2)(32-25)\text{°C} = 7.41\ \text{W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.85)(0.25+0.3\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(32+273\text{K})^4 - (25+273\text{K})^4] = 20.34\ \text{W}$$

Then the fraction of the heat loss from the outer surfaces of the box is determined to be

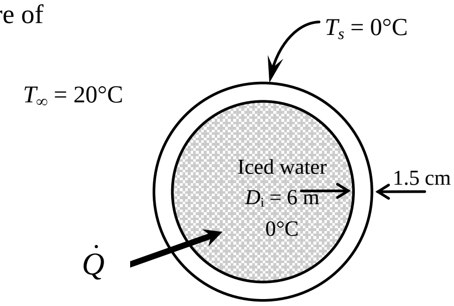
$$f = \frac{(6.55+7.41+20.34)\text{W}}{180\ \text{W}} = 0.1906 = \mathbf{19.1\%}$$

9-97 A spherical tank made of stainless steel is used to store iced water. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Thermal resistance of the tank is negligible. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (0+20)/2 = 10°C$ are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\upsilon = 1.426\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7336$$
$$\beta = \frac{1}{T_f} = \frac{1}{(10+273)\text{K}} = 0.003534 \text{K}^{-1}$$



Analysis (*a*) The characteristic length in this case is $L_c = D_o = 6.03$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D_o^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003534 \text{K}^{-1})(20-0 \text{ K})(6.03 \text{ m})^3}{(1.426\times10^{-5} \text{ m}^2/\text{s})^2}(0.7336) = 5.485\times10^{11}$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(0.469/\text{Pr}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(5.485\times10^{11})^{1/4}}{\left[1+\left(0.469/0.7336\right)^{9/16}\right]^{4/9}} = 394.5$$

$$h = \frac{k}{D_o}Nu = \frac{0.02439 \text{ W/m.°C}}{6.03 \text{ m}}(394.5) = 1.596 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D_o^2 = \pi(6.03 \text{ m})^2 = 114.2 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (1.596 \text{ W/m}^2\text{.°C})(114.2 \text{ m}^2)(20-0)\text{°C} = 3646 \text{ W}$$

Heat transfer by radiation and the total rate of heat transfer are

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (1)(114.2 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(20+273 \text{K})^4 - (0+273 \text{K})^4] = 11{,}759 \text{ W}$$
$$\dot{Q}_{total} = 3646 + 11{,}759 = 15{,}404 \text{ W} \cong \mathbf{15.4\ kW}$$

(*b*) The total amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (15.4 \text{ kJ/s})(24 \text{ h/day}\times3600 \text{s/h}) = 1.331\times10^6 \text{ kJ/day}$$

Then the amount of ice that melts during this period becomes

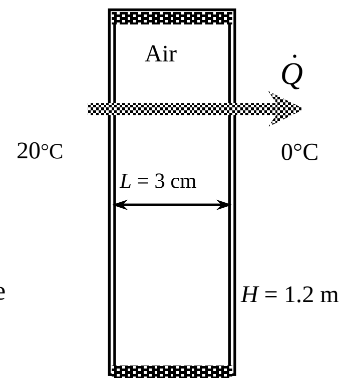
$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{1.331\times10^6 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{3988 kg}$$

9-98 A double-pane window consisting of two layers of glass separated by an air space is considered. The rate of heat transfer through the window and the temperature of its inner surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation effects are negligible. **4** The pressure of air inside the enclosure is 1 atm.

Properties We expect the average temperature of the air gap to be roughly the average of the indoor and outdoor temperatures, and evaluate The properties of air at 1 atm and the average temperature of $(T_{\infty 1}+T_{\infty 2})/2 = (20+0)/2 = 10°C$ are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\upsilon = 1.426\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7336$$
$$\beta = \frac{1}{T_f} = \frac{1}{(10+273)\text{K}} = 0.003534 \text{K}^{-1}$$



Analysis We "guess" the temperature difference across the air gap to be 15°C = 15 K for use in the Ra relation. The characteristic length in this case is the air gap thickness, $L_c = L = 0.03$ m. Then,

$$Ra = \frac{g\beta(T_1 - T_2)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003534 \text{ K}^{-1})(15 \text{ K})(0.03 \text{ m})^3}{(1.426\times10^{-5} \text{ m}^2\text{/s})^2}(0.7336) = 5.065\times10^4$$

Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = 0.42Ra^{1/4}\text{ Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(5.065\times10^4)^{1/4}(0.7336)^{0.012}\left(\frac{1.2 \text{ m}}{0.03 \text{ m}}\right)^{-0.3} = 2.076$$

$$h_{air} = \frac{k}{L}Nu = \frac{0.02439 \text{ W/m.°C}}{0.03 \text{ m}}(2.076) = 1.688 \text{ W/m}^2\text{.°C}$$

Then the rate of heat transfer through this double pane window is determined to be

$$A_s = H\times W = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

$$\dot{Q} = \frac{T_{\infty,i} - T_{\infty,o}}{R_{conv,i} + R_{cond,glasses} + R_{conv,air} + R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\dfrac{1}{h_i A_s} + \dfrac{2t_{\text{glass}}}{k_{glass} A_s} + \dfrac{1}{h_{air} A_s} + \dfrac{1}{h_o A_s}}$$

$$= \frac{20-0}{\dfrac{1}{(10)(2.4)} + \dfrac{2(0.003)}{(0.78)(2.4)} + \dfrac{1}{(1.688)(2.4)} + \dfrac{1}{(25)(2.4)}} = \mathbf{65\ W}$$

Check: The temperature drop across the air gap is determined from

$$\dot{Q} = hA_s\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{hA_s} = \frac{65 \text{ W}}{(1.688 \text{ W/m}^2\text{.°C})(2.4 \text{ m}^2)} = 16.0°C$$

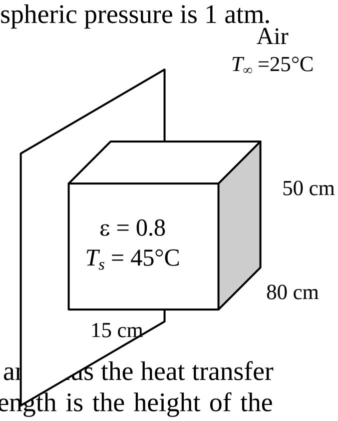
which is very close to the assumed value of 15°C used in the evaluation of the Ra number.

9-99 An electric resistance space heater filled with oil is placed against a wall. The power rating of the heater and the time it will take for the heater to reach steady operation when it is first turned on are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the back, bottom, and top surfaces are disregarded. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (45+25)/2 = 35°C$ are (Table A-15)

$$k = 0.02625 \text{ W/m.°C}$$
$$\upsilon = 1.655\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7268$$
$$\beta = \frac{1}{T_f} = \frac{1}{(35+273)\text{K}} = 0.003247 \text{K}^{-1}$$



Analysis Heat transfer from the top and bottom surfaces are said to be negligible, an[illegible]as the heat transfer area in this case consists of the three exposed side surfaces. The characteristic length is the height of the box, $L_c = L = 0.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003247 \text{K}^{-1})(45-25 \text{ K})(0.5 \text{ m})^3}{(1.655\times10^{-5} \text{ m}^2/\text{s})^2}(0.7268) = 2.114\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.114\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7268}\right)^{9/16}\right]^{8/27}}\right\}^2 = 76.68$$

$$h = \frac{k}{L}Nu = \frac{0.02625 \text{ W/m.°C}}{0.5 \text{ m}}(76.68) = 4.026 \text{ W/m}^2.°\text{C}$$
$$A_s = (0.5 \text{ m})(0.8 \text{ m}) + 2(0.15 \text{ m})(0.5 \text{ m}) = 0.55 \text{ m}^2$$

and
$$\dot{Q} = hA_s(T_s - T_\infty) = (4.026 \text{ W/m}^2.°\text{C})(0.55 \text{ m}^2)(45-25)°\text{C} = 44.3 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) = (0.8)(0.55 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(45+273 \text{K})^4 - (25+273 \text{K})^4] = 58.4 \text{ W}$$

Then the total rate of heat transfer, thus the power rating of the heater becomes

$$\dot{Q}_{total} = 44.3 + 58.4 = \mathbf{102.7\ W}$$

The specific heat of the oil at the average temperature of the oil is 1943 J/kg.°C. Then the amount of heat transfer needed to raise the temperature of the oil to the steady operating temperature and the time it takes become

$$Q = mC_p(T_2 - T_1) = (45 \text{ kg})(1943 \text{ J/kg.°C})(45-25)°\text{C} = 1.749\times10^6 \text{ J}$$

$$Q = \dot{Q}\Delta t \longrightarrow \Delta t = \frac{Q}{\dot{Q}} = \frac{1.749\times10^6 \text{ kJ}}{100 \text{ J/s}} = 17{,}034 \text{ s} = \mathbf{4.73\ h}$$

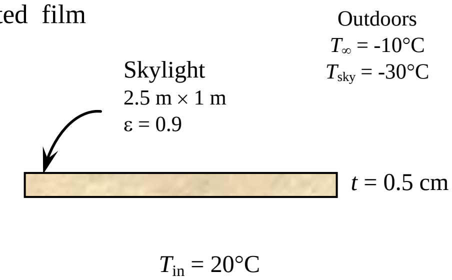
which is not practical. Therefore, the surface temperature of the heater must be allowed to be higher than 45°C.

9-100 A horizontal skylight made of a single layer of glass on the roof of a house is considered. The rate of heat loss through the skylight is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (-4-10)/2 = -7°C$ are (Table A-15)

$$k = 0.0231 \text{ W/m.°C}$$
$$\upsilon = 1.278\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.738$$
$$\beta = \frac{1}{T_f} = \frac{1}{(-7+273)\text{K}} = 0.003759 \text{K}^{-1}$$



Analysis We assume radiation heat transfer inside the house to be negligible. We start the calculations by "guessing" the glass temperature to be 4°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is determined from $L_c = \frac{A_s}{p} = \frac{(1\text{m})(2.5\text{m})}{2(1\text{m}+2.5\text{m})} = 0.357\text{m}$. Then,

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{m/s}^2)(0.003759\text{K}^{-1})[-4-(-10)\text{ K}](0.357\text{m})^3}{(1.278\times10^{-5}\text{ m}^2\text{/s})^2}(0.738) = 4.553\times10^7$$

$$Nu = 0.15Ra^{1/3} = 0.15(4.553\times10^7)^{1/3} = 53.56$$

$$h_o = \frac{k}{L_c}Nu = \frac{0.0231\text{W/m.°C}}{0.357\text{m}}(53.56) = 3.465\text{W/m}^2\text{.°C}$$

$$A_s = (1\text{m})(2.5\text{m}) = 2.5\text{m}^2$$

Using the assumed value of glass temperature, the radiation heat transfer coefficient is determined to be

$$\begin{aligned}h_{rad} &= \varepsilon\sigma(T_s + T_{sky})(T_s{}^2 + T_{sky}{}^2) \\ &= 0.9(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(-4+273)+(-30+273)][(-4+273)^2+(-30+273)^2]\text{K}^3 \\ &= 3.433\text{W/m}^2\text{.K}\end{aligned}$$

Then the combined convection and radiation heat transfer coefficient outside becomes

$$h_{o,combined} = h_o + h_{rad} = 3.465 + 3.433 = 6.898\text{W/m}^2$$

Again we take the glass temperature to be -4°C for the evaluation of the properties and *h* for the inner surface of the skylight. The properties of air at 1 atm and the film temperature of $T_f = (-4+20)/2 = 8°C$ are (Table A-15)

$$k = 0.02424\text{W/m.°C}$$
$$\upsilon = 1.409\times10^{-5}\text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7342$$
$$\beta = \frac{1}{T_f} = \frac{1}{(8+273)\text{K}} = 0.003559\text{K}^{-1}$$

The characteristic length in this case is also 0.357 m. Then,

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\upsilon^2}\Pr = \frac{(9.81\,\text{m/s}^2)(0.003559\,\text{K}^{-1})[20-(-4)\,\text{K})(0.357\,\text{m})^3}{(1.409\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7342) = 1.412\times10^8$$

$$Nu = 0.27Ra^{1/4} = 0.27(1.412\times10^8)^{1/4} = 29.43$$

$$h_i = \frac{k}{L_c}Nu = \frac{0.02424\,\text{W/m.°C}}{0.357\,\text{m}}(29.43) = 1.998\,\text{W/m}^2\text{.°C}$$

Using the thermal resistance network, the rate of heat loss through the skylight is determined to be

$$\dot{Q}_{skylight} = \frac{T_{s,i} - T_{\infty,o}}{R_{conv,i} + R_{cond,glas} + R_{combined,o}}$$

$$= \frac{A_s(T_{room} - T_{out})}{\dfrac{1}{h_i} + \dfrac{t_{\text{glass}}}{k_{glass}} + \dfrac{1}{h}} = \frac{(2.5\,\text{m}^2)[20-(-10)]°\text{C}}{\dfrac{1}{1.998\,\text{W/m}^2\text{.°C}} + \dfrac{0.005\,\text{m}}{0.78\,\text{W/m.°C}} + \dfrac{1}{6.898\,\text{W/m}^2\text{.°C}}} = 115\,\text{W}$$

Using the same heat transfer coefficients for simplicity, the rate of heat loss through the roof in the case of R-5.34 construction is determined to be

$$\dot{Q}_{roof} = \frac{T_{s,i} - T_{\infty,o}}{R_{conv,i} + R_{cond} + R_{combined,o}}$$

$$= \frac{A_s(T_{room} - T_{out})}{\dfrac{1}{h_i} + R_{glass} + \dfrac{1}{h}} = \frac{(2.5\,\text{m}^2)[20-(-10)]°\text{C}}{\dfrac{1}{1.998\,\text{W/m}^2\text{.°C}} + 5.34\,\text{m}^2\text{.°C/W} + \dfrac{1}{6.898\,\text{W/m}^2\text{.°C}}} = 5.36\,\text{W}$$

Therefore, a house loses 115/5.36 ≅ 21 times more heat through the skylights than it does through an insulated wall of the same size.

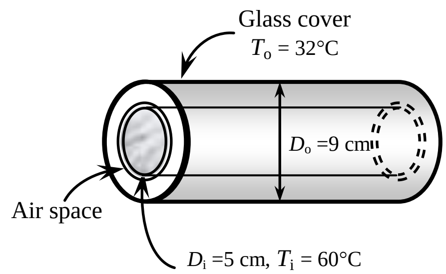
Using Newton's law of cooling, the glass temperature corresponding to a heat transfer rate of 115 W is calculated to be –3.3°C, which is sufficiently close to the assumed value of -4°C. Therefore, there is no need to repeat the calculations.

9-101 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. Water is heated in the tube, and the annular space between the copper and glass tube is filled with air. The rate of heat loss from the collector by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation effects are negligible. **3** The pressure of air in the enclosure is 1 atm.

Properties The properties of air at 1 atm and the average temperature of $(T_i+T_o)/2$ = (60+32)/2 = 46°C are (Table A-15)

$$k = 0.02706\ \text{W/m.°C}$$
$$\upsilon = 1.759\times10^{-5}\ \text{m}^2/\text{s}$$
$$\text{Pr} = 0.7239$$
$$\beta = \frac{1}{T_f} = \frac{1}{(46+273)\text{K}} = 0.003135\ \text{K}^{-1}$$



Analysis The characteristic length in this case is the distance between the two cylinders ,

$$L_c = \frac{D_o - D_i}{2} = \frac{(9-5)\ \text{cm}}{2} = 2\ \text{cm}$$

and,

$$Ra = \frac{g\beta(T_i - T_o)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003135\ \text{K}^{-1})(60-32\ \text{K})(0.02\ \text{m})^3}{(1.759\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7239) = 16{,}106$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.09\ \text{m}}{0.05\ \text{m}}\right]^4}{(0.02\ \text{m})^3\left[(0.05\ \text{m})^{-7/5}+(0.09\ \text{m})^{-7/5}\right]^5} = 0.1303$$

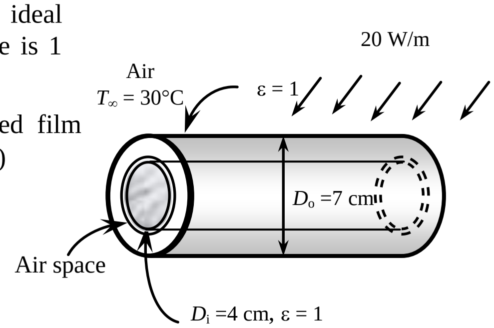
$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$
$$= 0.386(0.02706\ \text{W/m.°C})\left(\frac{0.7239}{0.861+0.7239}\right)^{1/4}\left[(0.1303)(16{,}106)\right]^{1/4} = 0.05812\ \text{W/m.°C}$$

Then the heat loss from the collector per meter length of the tube becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_i - T_o) = \frac{2\pi(0.05812\ \text{W/m.°C})}{\ln\left(\dfrac{0.09\ \text{m}}{0.05\ \text{m}}\right)}(60-32)\text{°C} = \mathbf{17.4\ W}$$

9-102 A solar collector consists of a horizontal tube enclosed in a concentric thin glass tube is considered. The pump circulating the water fails. The temperature of the aluminum tube when equilibrium is established is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (33+30)/2 = 31.5°C are (Table A-15)

$$k = 0.02599 \text{ W/m.°C}$$
$$\upsilon = 1.622\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7278$$
$$\beta = \frac{1}{T_f} = \frac{1}{(31.5+273)\text{K}} = 0.003284 \text{K}^{-1}$$

Analysis This problem involves heat transfer from the aluminum tube to the glass cover, and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfers will be equal to the rate of heat gain. That is,

$$\dot{Q}_{tube-glass} = \dot{Q}_{glass-ambient} = \dot{Q}_{solar\ gain} = 20 \text{ W} \ \ \text{(per meter length)}$$

Now we assume the surface temperature of the glass cover to be 33°C. We will check this assumption later on, and repeat calculations with a better assumption, if necessary.

The characteristic length for the outer diameter of the glass cover $L_c = D_o$ =0.07 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D_o^3}{\upsilon^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003284 \text{K}^{-1})(33-30 \text{ K})(0.07 \text{ m})^3}{(1.622\times10^{-5} \text{ m}^2\text{/s})^2}(0.7278) = 91{,}679$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(91{,}679)^{1/6}}{\left[1+\left(0.559/0.7278\right)^{9/16}\right]^{8/27}}\right\}^2 = 7.626$$

$$A_s = \pi D_o L = \pi(0.07 \text{ m})(1 \text{ m}) = 0.2199 \text{ m}^2$$

$$h = \frac{k}{D_o}Nu = \frac{0.02599 \text{ W/m.°C}}{0.07 \text{ m}}(7.626) = 2.832 \text{ W/m}^2\text{.°C}$$

and,

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (2.832 \text{ W/m}^2\text{.°C})(0.2199 \text{ m}^2)(T_{glass} - 30)\text{°C}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4) = (1)(0.2199 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{glass}+273 \text{ K})^4 - (20+273 \text{ K})^4\right]$$

The expression for the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$20 \text{ W} = (2.832 \text{ W/m}^2\text{.°C})(0.2199 \text{ m}^2)(T_{glass} - 30)\text{°C}$$
$$+ (1)(0.2199 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{glass}+273 \text{ K})^4 - (20+273 \text{ K})^4\right]$$

Its solution is $T_{glass} = 33.34\text{°C}$, which is sufficiently close to the assumed value of 33°C.

Now we will calculate heat transfer through the air layer between aluminum tube and glass cover. We will assume the aluminum tube temperature to be 45°C and evaluate properties at the average temperature of

$(T_i+T_o)/2 = (45+33.34)/2 = 39.17°C$ are (Table A-15)

$$k = 0.02656 \text{ W/m.°C}$$
$$\upsilon = 1.694\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7257$$
$$\beta = \frac{1}{T_f} = \frac{1}{(39.17+273)\text{K}} = 0.003203 \text{K}^{-1}$$

The characteristic length in this case is the distance between the two cylinders,

$$L_c = (D_o - D_i)/2 = (7-4)/2 \text{ cm} = 1.5 \text{ cm}$$

Then,

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003203 \text{K}^{-1})(45-33.34 \text{ K})(0.015 \text{ m})^3}{(1.694\times10^{-5} \text{ m}^2\text{/s})^2}(0.7257) = 3125$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.07 \text{ m}}{0.04 \text{ m}}\right]^4}{(0.015 \text{ m})^3\left[(0.04 \text{ m})^{-7/5}+(0.07 \text{ m})^{-7/5}\right]^5} = 0.1254$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$
$$= 0.386(0.02656 \text{ W/m.°C})\left(\frac{0.7257}{0.861+0.7257}\right)^{1/4}\left[(0.1254)(3125)\right]^{1/4} = 0.03751 \text{ W/m.°C}$$

The heat transfer expression is

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_1-T_2) = \frac{2\pi(0.03751 \text{ W/m.°C})}{\ln\left(\dfrac{0.07 \text{ m}}{0.04 \text{ m}}\right)}(T_{tube}-33.34)\text{°C}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) = (1)(0.1684 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{tube}+273 \text{K})^4-(33.34+273 \text{K})^4\right]$$

The expression for the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$20 \text{ W} = \frac{2\pi(0.03751 \text{ W/m.°C})}{\ln\left(\dfrac{0.07 \text{ m}}{0.04 \text{ m}}\right)}(T_{tube}-33.34)\text{°C}$$
$$+(1)(0.1684 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{tube}+273 \text{K})^4-(33.34+273 \text{K})^4\right]$$

Its solution is $T_{tube} = \mathbf{45.9°C}$,

which is sufficiently close to the assumed value of 45°C. Therefore, there is no need to repeat the calculations.