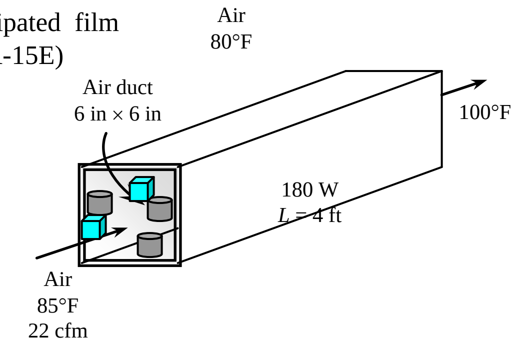
9-103E The components of an electronic device located in a horizontal duct of rectangular cross section is cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (120+80)/2 = 100°F are (Table A-15E)



$$k = 0.01529 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.1808\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.726$$
$$\beta = 1/T_f = 1/(100+460)\text{R} = 0.001786\text{R}^{-1}$$

Analysis (*a*) Using air properties at the average temperature of (85+100))/2 = 92.5°F and 1 atm for the forced air, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho\dot{V} = (0.07186 \text{ lbm/ft}^3)(22 \text{ ft}^3\text{/min}) = 1.581 \text{ lbm/min}$$

$$\dot{Q}_{forced} = \dot{m}C_p(T_{out} - T_{in}) = (1.581\times 60 \text{ lbm/h})(0.2405 \text{ Btu/lbm.°F})(100-85)\text{°F} = 342.1 \text{ Btu/h}$$

Noting that radiation heat transfer is negligible, the rest of the 180 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} - \dot{Q}_{forced} = (180\times3.412) - 342.1 = \mathbf{272\ Btu/h}$$

(*b*) We start the calculations by "guessing" the surface temperature to be 120°F for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary.

Horizontal top surface: The characteristic length is $L_c = \dfrac{A_s}{P} = \dfrac{(4\text{ ft})(6/12\text{ft})}{2(4\text{ ft}+6/12\text{ ft})} = 0.2222\text{ ft}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)(0.001786\text{R}^{-1})(120-80\text{ R})(0.2222\text{ ft})^3}{(0.1808\times10^{-3}\text{ ft}^2\text{/s})^2}(0.726) = 5.604\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(5.604\times10^5)^{1/4} = 14.77$$

$$h_{top} = \frac{k}{L_c}Nu = \frac{0.01529\text{ Btu/h.ft.°F}}{0.2222\text{ ft}}(14.77) = 1.016\text{ Btu/h.ft}^2\text{.°F}$$

$$A_{top} = (4\text{ ft})(6/12\text{ ft}) = 2\text{ ft}^2 = A_{bottom}$$

Horizontal bottom surface: The Nusselt number for this geometry and orientation can be determined from

$$Nu = 0.27Ra^{1/4} = 0.27(5.604\times10^5)^{1/4} = 7.387$$

$$h_{bottom} = \frac{k}{L_c}Nu = \frac{0.01529\text{ Btu/h.ft.°F}}{0.2222\text{ ft}}(7.387) = 0.5082\text{ Btu/h.ft}^2\text{.°F}$$

Vertical side surfaces: The characteristic length in this case is the height of the duct, $L_c = L$ = 6 in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)(0.001786\text{R}^{-1})(120-80\text{ R})(0.5\text{ ft})^3}{(0.1808\times10^{-3}\text{ ft}^2\text{/s})^2}(0.726) = 6.383\times10^6$$

$$Nu = \left\{ 0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}} \right\}^2 = \left\{ 0.825 + \frac{0.387(6.383\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.726}\right)^{9/16}\right]^{8/27}} \right\}^2 = 27.57$$

$$h_{side} = \frac{k}{L} Nu = \frac{0.01529\,\text{Btu/h.ft.°F}}{0.5\,\text{ft}}(27.57) = 0.843\,\text{Btu/h.ft}^2\text{.°F}$$

$$A_{side} = 2(4\,\text{ft})(0.5\,\text{ft}) = 4\,\text{ft}^2$$

Then the total heat loss from the duct can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{side} = [(hA)_{top} + (hA)_{bottom} + (hA)_{side}](T_s - T_\infty)$$

Substituting and solving for the surface temperature,

$$272\,\text{Btu/h} = [(1.016\times2 + 0.5082\times2 + 0.843\times4)\,\text{Btu/h.°F}](T_s - 80)\text{°F}$$

T_s = **122.4°F**

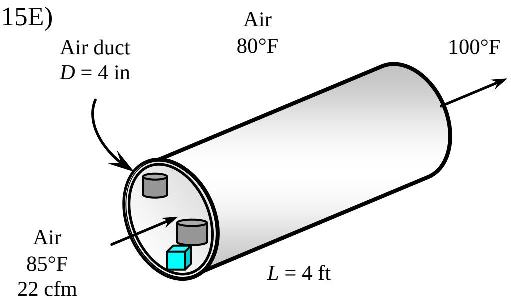
which is sufficiently close to the assumed value of 120°F used in the evaluation of properties and *h*. Therefore, there is no need to repeat the calculations.

9-104E The components of an electronic system located in a horizontal duct of circular cross section is cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (150+80)/2 = 115$°F are (Table A-15E)

$$k = 0.01564 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.1894\times10^{-3}\text{ ft}^2/\text{s}$$
$$\Pr = 0.7268$$
$$\beta = \frac{1}{T_f} = \frac{1}{(115+460)\text{ R}} = 0.001739\text{R}^{-1}$$



Analysis (a) Using air properties at the average temperature of (85+100))/2 = 92.5°F and 1 atm for the forced air, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho\dot{V} = (0.07186\text{ lbm/ft}^3)(22\text{ ft}^3/\text{min}) = 1.581\text{ lbm/min}$$

$$\dot{Q}_{forced} = \dot{m}C_p(T_{out}-T_{in}) = (1.581\times60\text{ lbm/h})(0.2405\text{ Btu/lbm.°F})(100-85)\text{°F} = 342.1\text{ Btu/h}$$

Noting that radiation heat transfer is negligible, the rest of the 180 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} - \dot{Q}_{forced} = (180\times3.412) - 342.1 = \mathbf{272\ Btu/h}$$

(b) We start the calculations by "guessing" the surface temperature to be 150°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the duct, $L_c = D = 4$ in. Then,

$$Ra = \frac{g\beta(T_1-T_2)D^3}{\upsilon^2}\Pr = \frac{(32.2\text{ ft/s}^2)(0.001739\text{R}^{-1})(150-80\text{ R})(4/12\text{ ft})^3}{(0.1894\times10^{-3}\text{ ft}^2/\text{s})^2}(0.7268) = 2.930\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.930\times10^6)^{1/6}}{\left[1+(0.559/0.7268)^{9/16}\right]^{8/27}}\right\}^2 = 19.79$$

$$h = \frac{k}{D}Nu = \frac{0.01564\text{ Btu/h.ft.°F}}{4/12\text{ ft}}(19.79) = 0.9287\text{ Btu/h.ft}^2.\text{°F}$$

$$A_s = \pi DL = \pi(4/12\text{ ft})(4\text{ ft}) = 4.19\text{ ft}^2$$

Then the surface temperature is determined to be

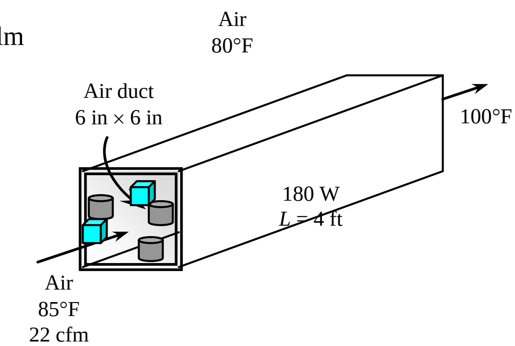
$$\dot{Q} = hA_s(T_s - T_\infty) \rightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 80\text{°F} + \frac{272\text{ Btu/h}}{(0.9287\text{ Btu/h.ft}^2.\text{°F})(4.19\text{ ft}^2)} = \mathbf{149.9°F}$$

which is practically equal to the assumed value of 150°F used in the evaluation of properties and h. Therefore, there is no need to repeat the calculations.

9-105E The components of an electronic system located in a horizontal duct of rectangular cross section is cooled by natural convection. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (160+80)/2 = 120°F are (Table A-15E)



$$k = 0.01576\ \text{Btu/h.ft.°F}$$
$$\upsilon = 0.1923\times10^{-3}\ \text{ft}^2/\text{s}$$
$$\text{Pr} = 0.723$$
$$\beta = 1/T_f = 1/(120+460\ \text{R}) = 0.001724\ \text{R}^{-1}$$

Analysis (a) Noting that radiation heat transfer is negligible and no heat is removed by forced convection because of the failure of the fan, the entire 180 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} = \mathbf{180\ W}$$

(b) We start the calculations by "guessing" the surface temperature to be 160°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary.

Horizontal top surface: The characteristic length is $L_c = \dfrac{A_s}{p} = \dfrac{(4\ \text{ft})(6/12\text{ft})}{2(4\ \text{ft}+6/12\ \text{ft})} = 0.2222\ \text{ft}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} = \frac{(32.2\ \text{ft/s}^2)(0.001724\text{R}^{-1})(160-80\ \text{R})(0.2222\ \text{ft})^3}{(0.1923\times10^{-3}\ \text{ft}^2/\text{s})^2}(0.723) = 9.534\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(9.534\times10^5)^{1/4} = 16.87$$

$$h_{top} = \frac{k}{L_c}Nu = \frac{0.01576\ \text{Btu/h.ft.°F}}{0.2222\ \text{ft}}(16.87) = 1.197\ \text{Btu/h.ft}^2.\text{°F}$$

$$A_{top} = (4\ \text{ft})(6/12\ \text{ft}) = 2\ \text{ft}^2 = A_{bottom}$$

Horizontal bottom surface: The Nusselt number for this geometry and orientation can be determined from

$$Nu = 0.27Ra^{1/4} = 0.27(9.534\times10^5)^{1/4} = 8.437$$

$$h_{bottom} = \frac{k}{L_c}Nu = \frac{0.01576\ \text{Btu/h.ft.°F}}{0.2222\ \text{ft}}(8.437) = 0.5983\ \text{Btu/h.ft}^2.\text{°F}$$

Vertical side surfaces: The characteristic length in this case is the height of the duct, $L_c = L$ = 6 in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(32.2\ \text{ft/s}^2)(0.001724\text{R}^{-1})(160-80\ \text{R})(0.5\ \text{ft})^3}{(0.1923\times10^{-3}\ \text{ft}^2/\text{s})^2}(0.723) = 1.086\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.086\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.723}\right)^{9/16}\right]^{8/27}}\right\}^2 = 32.03$$

$$h_{side} = \frac{k}{L}Nu = \frac{0.01576\ \text{Btu/h.ft.°F}}{0.5\ \text{ft}}(32.03) = 1.009\ \text{Btu/h.ft}^2.\text{°F}$$

$$A_{side} = 2(4\ \text{ft})(0.5\ \text{ft}) = 4\ \text{ft}^2$$

Then the total heat loss from the duct can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{side} = [(hA)_{top} + (hA)_{bottom} + (hA)_{side}](T_s - T_\infty)$$

Substituting and solving for the surface temperature,

$$180\text{ W}\left(\frac{3.41214\text{Btu/h}}{1\text{ W}}\right) = [(1.197\times 2 + 0.5983\times 2 + 1.009\times 4)\text{ Btu/h.°F}](T_s - 80)\text{°F}$$

T_s = **160.5°F**

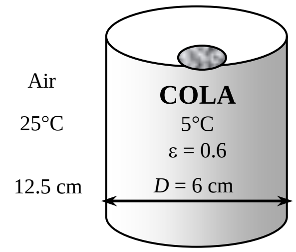
which is sufficiently close to the assumed value of 160°F used in the evaluation of properties and *h*. Therefore, there is no need to repeat the calculations.

9-106 A cold aluminum canned drink is exposed to ambient air. The time it will take for the average temperature to rise to a specified value is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the bottom surface of the can is disregarded. **5** The thermal resistance of the can is negligible.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (6+25)/2 = 15.5°C are (Table A-15)

$$k = 0.0248 \text{ W/m.°C}$$
$$\upsilon = 1.475\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7321$$
$$\beta = 1/T_f = 1/(15.5+273\text{K}) = 0.003466\text{K}^{-1}$$



Analysis We assume the surface temperature of aluminum can to be equal to the temperature of the drink in the can since the can is made of a very thin layer of aluminum. Noting that the temperature of the drink rises from 5°C to 7°C, we take the average surface temperature to be 6°C. The characteristic length in this case is the height of the box $L_c = L$ = 0.125 m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003466\text{K}^{-1})(25-6\text{ K})(0.125\text{ m})^3}{(1.475\times10^{-5}\text{ m}^2\text{/s})^2}(0.7321) = 4.246\times10^6$$

At this point we should check if we can treat this aluminum can as a vertical plate. The criteria is

$$D \ge \frac{35L}{Gr^{1/4}} \longrightarrow \frac{35(12.5\text{ cm})}{(4.246\times10^6/0.7321)^{1/4}} = 9.92\text{ cm}$$

which is not smaller than the diameter of the can (6 cm), but close to it. Therefore, we can still use vertical plate relation approximately (besides, we do not have another relation available). Then the Nusselt number becomes from

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(4.246\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7321}\right)^{9/16}\right]^{8/27}}\right\}^2 = 23.81$$

$$h = \frac{k}{L}Nu = \frac{0.0248\text{ W/m.°C}}{0.125\text{ m}}(23.81) = 4.887\text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL + \frac{\pi D^2}{4} = \pi(0.06\text{ m})(0.125\text{ m}) + \frac{\pi(0.06\text{ m})^2}{4} = 0.02639\text{m}^2$$

Note that we also include top surface area of the can to the total surface area, and assume the heat transfer coefficient for that area to be the same for simplicity (actually, it will be a little lower). Then heat transfer rate from outer surfaces of the can by natural convection becomes

$$\dot{Q} = hA_s(T_\infty - T_s) = (4.887\text{ W/m}^2\text{.°C})(0.02639\text{m}^2)(25-6)\text{°C} = 2.45\text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)$$
$$= (0.6)(0.02639\text{m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(25+273\text{K})^4 - (6+273\text{K})^4] = 1.64\text{ W}$$

and $\dot{Q}_{total} = 2.41 + 1.64 = 4.09\text{ W}$

Using the properties of water for the cold drink at 6°C, the amount of heat transfer to the drink is determined from

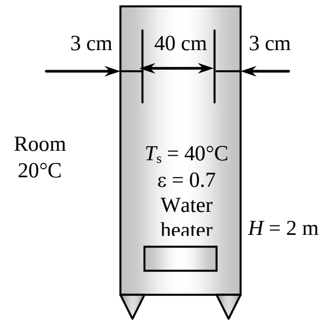
$$m = \rho V = \rho\frac{\pi D^2}{4}L = (1000\text{ kg/m}^3)\frac{\pi(0.06\text{ m})^2}{4}(0.125\text{m}) = 0.3534\text{ kg}$$
$$Q = mC_p(T_2 - T_1) = (0.353\text{ kg})(4197\text{ J/kg.°C})(7-5)\text{°C} = 2967\text{ J}$$

Then the time required for the temperature of the cold drink to rise to 7°C becomes

$$Q = \dot{Q}\Delta t \longrightarrow \Delta t = \frac{Q}{\dot{Q}} = \frac{2967\,\text{J}}{4.09\,\text{J/s}} = 725\,\text{s} = \mathbf{12.1\,min}$$

9-107 An electric hot water heater is located in a small room. A hot water tank insulation kit is available for $30. The payback period of this insulation to pay for itself from the energy it saves is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the top and bottom surfaces of the tank is disregarded. **5** The thermal resistance of the metal sheet is negligible.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (40+20)/2 = 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\upsilon = 1.608\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7282$$
$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033\text{K}^{-1}$$

Analysis The characteristic length in this case is the height of the heater, $L_c = L$ = 2 m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.0033\text{K}^{-1})(40-20\text{ K})(2\text{ m})^3}{(1.608\times10^{-5}\text{ m}^2/\text{s})^2}(0.7282) = 1.459\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.459\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 285.4$$

$$h = \frac{k}{L}Nu = \frac{0.02588 \text{ W/m.°C}}{2\text{ m}}(285.4) = 3.693 \text{ W/m}^2.°\text{C}$$
$$A_s = \pi DL = \pi(0.46\text{ m})(2\text{ m}) = 2.89\text{ m}^2$$

and $\dot{Q} = hA_s(T_\infty - T_s) = (3.693 \text{ W/m}^2.°\text{C})(2.89\text{ m}^2)(40-20)°\text{C} = 213.5\text{ W}$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4) = (0.7)(2.89\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(40+273\text{K})^4 - (20+273\text{K})^4] = 255.6\text{ W}$$

and $\dot{Q}_{total} = 213.5 + 255.6 = 469\text{ W}$

The reduction in heat loss after adding insulation is

$$\dot{Q} = (0.80)(469\text{ W}) = 375.2\text{ W}$$

The amount of heat and money saved per hour is

$$Q_{saved} = \dot{Q}_{saved}\Delta t = (0.3752\text{ kW})(1\text{h}) = 0.3752\text{ kWh}$$

$$\text{Money saved} = (0.3752\text{ kWh})(\$0.08/\text{kWh}) = \$0.03002$$

Then it will take

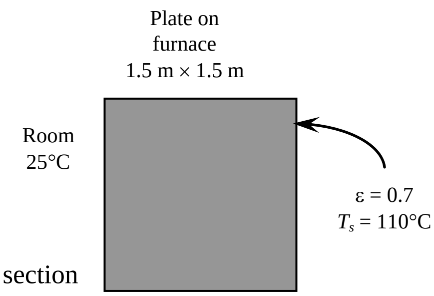
$$\Delta t = \frac{\$30}{\$0.03002} = 999.4\text{ h} = \mathbf{41.64\ days}$$

for the additional insulation to pay for itself from the energy it saves.

9-108 A hot part of the vertical front section of a natural gas furnace in a plant is considered. The rate of heat loss from this section and the annual cost of this heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from other surfaces of the tank is disregarded.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (110+25)/2 = 67.5°C$ are (Table A-15)



$$k = 0.02863 \text{ W/m.°C}$$
$$\upsilon = 1.97\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7184$$
$$\beta = \frac{1}{T_f} = \frac{1}{(67.5+273)\text{K}} = 0.002937 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the height of that section of furnace, $L_c = L = 15$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002937 \text{ K}^{-1})(110-25 \text{ K})(1.5 \text{ m})^3}{(1.97\times10^{-5} \text{ m}^2\text{/s})^2}(0.7184) = 1.530\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.530\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7184}\right)^{9/16}\right]^{8/27}}\right\}^2 = 289.1$$

$$h = \frac{k}{L}Nu = \frac{0.02863 \text{ W/m.°C}}{1.5 \text{ m}}(289.1) = 5.518 \text{ W/m}^2\text{.°C}$$
$$A_s = (1 \text{ m})(1.5 \text{ m}) = 1.5 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.518 \text{ W/m}^2\text{.°C})(1.5 \text{ m}^2)(110-25)\text{°C} = 703.5 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_{surr}{}^4 - T_s{}^4)$$
$$= (0.7)(1.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(110+273\text{K})^4 - (25+273\text{K})^4] = 812 \text{ W}$$

$$\dot{Q}_{total} = 703.5 + 812 = \mathbf{1515 \text{ W}}$$

The amount and cost of natural gas used to overcome this heat loss per year is

$$Q_{gas} = \dot{Q}_{gas}\Delta t = \frac{\dot{Q}_{total}}{0.79}\Delta t = \frac{(1.5151 \text{ kJ/s})}{0.79}(310 \text{ days/yr}\times10 \text{ hr/day}\times3600 \text{ s/hr}) = 2.14\times10^7 \text{ kJ}$$

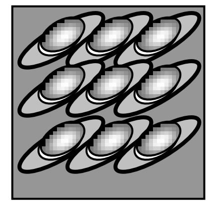
$$\text{Cost} = (2.14\times10^7 / 105{,}500 \text{ therm})(\$075\text{/therm}) = \mathbf{\$152.2}$$

9-109 A group of 25 transistors are cooled by attaching them to a square aluminum plate and mounting the plate on the wall of a room. The required size of the plate to limit the surface temperature to 50°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the back side of the plate is negligible.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40$°C are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\upsilon = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{K}^{-1}$$



Analysis The Rayleigh number can be determined in terms of the characteristic length (length of the plate) to be

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(L)^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 1.571\times10^9 L^3$$

The Nusselt number relation is

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.571\times10^9 L^3)^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2$$

The heat transfer coefficient is

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{L}Nu$$
$$A_s = L^2$$

Noting that both the surface and surrounding temperatures are known, the rate of convection and radiation heat transfer are expressed as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = \frac{0.02662 \text{ W/m.°C}}{L}NuL^2(50-30)\text{°C}$$
$$\dot{Q}_{\text{rad}} = \varepsilon A_s\sigma(T_s{}^4 - T_{sky}{}^4) = (0.9)L^2(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(50+273)^4 - (30+273)^4]\text{K}^4 = 125.3L^2$$

The rate of total heat transfer is expressed as

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$
$$25\times(1.5 \text{ W}) = \frac{0.02662 \text{ W/m.°C}}{L}NuL^2(50-30)\text{°C} + 125.3L^2$$

Substituting Nusselt number expression above into this equation and solving for L, the length of the plate is determined to be

L = **0.426 m**

9-110 A group of 25 transistors are cooled by attaching them to a square aluminum plate and positioning the plate horizontally in a room. The required size of the plate to limit the surface temperature to 50°C is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the back side of the plate is negligible.

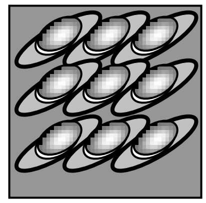
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$

$$\upsilon = 1.702\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7255$$

$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{K}^{-1}$$



Analysis The characteristic length and the Rayleigh number for the horizontal case are determined to be

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4}$$

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{m/s}^2)(0.003195 \text{K}^{-1})(50-30 \text{K})(L/4)^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 2.454\times10^7 L^3$$

Noting that both the surface and surrounding temperatures are known, the rate of radiation heat transfer is determined to be

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma(T_s^{\ 4} - T_{sky}^{\ 4}) = (0.9)L^2(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(50+273)^4 - (30+273)^4]\text{K}^4 = 125.3L^2$$

(*a*) **Hot surface facing up:** We assume Ra < 10^7 and thus L <0.74 m so that we can determine the Nu number from Eq. 9-22. Then the Nusselt number and the convection heat transfer coefficient become

$$Nu = 0.54Ra^{1/4} = 0.54(2.454\times10^7 L^3)^{1/4} = 38.0L^{3/4}$$

Then,

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{W/m.°C}}{L/4}(38.0L^{3/4}) = 4.047L^{-1/4} \text{ W/m}^2.\text{°C}$$

$$A_s = L^2$$

The rate of convection heat transfer is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (4.047L^{-1/4})L^2(50-30) = 80.94L^{7/8} \text{ W}$$

Then,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$25\times(1.5 \text{ W}) = 80.94L^{7/4} + 125.3L^2 \text{ W}$$

Solving for L, the length of the plate is determined to be

L = **0.407 m**

Note that L < 0.75 m, and therefore the assumption of Ra < 10^7 is verified. That is,

(*b*) **Hot surface facing down:** The Nusselt number in this case is determined from

$$Nu = 0.27Ra^{1/4} = 0.27(2.454\times10^7 L^3)^{1/4} = 19.0L^{3/4}$$

Then,

$$h = \frac{k}{L_c}Nu = \frac{0.02662 \text{W/m.°C}}{L/4}(19.0L^{3/4}) = 2.023L^{-1/4}$$

The rate of convection heat transfer is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (2.023L^{-1/4})L^2(50-30) = 40.47L^{7/8} \quad \text{W}$$

Then,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$25 \times (1.5 \text{ W}) = 40.47L^{7/8} + 125.3L^2 \text{ W}$$

Solving for *L*, the length of the plate is determined to be

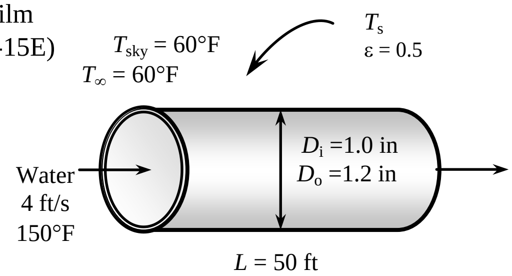
L = **0.464 m**

9-111E A hot water pipe passes through a basement. The temperature drop of water in the basement due to heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (150+60)/2 = 105°F are (Table A-15E)

$$k = 0.01541 \text{ Btu/h.ft.°F}$$
$$\upsilon = 0.1837\times10^{-3} \text{ ft}^2\text{/s}$$
$$\Pr = 0.7253$$
$$\beta = \frac{1}{T_f} = \frac{1}{(105+460)\text{R}} = 0.00177\text{R}^{-1}$$



Analysis We expect the pipe temperature to be very close to the water temperature, and start the calculations by "guessing" the average outer surface temperature of the pipe to be 150°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the pipe, $L_c = D_o$ = 1.2 in. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D_o^3}{\upsilon^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.00177\text{R}^{-1})(150-60\text{ R})(1.2/12\text{ ft})^3}{(0.1837\times10^{-3}\text{ ft}^2\text{/s})^2}(0.7253) = 1.102\times10^5$$

The natural convection Nusselt number can be determined from

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.1023\times10^5)^{1/6}}{\left[1+\left(0.559/0.7253\right)^{9/16}\right]^{8/27}}\right\}^2 = 7.999$$

$$h_o = \frac{k}{D_o}Nu = \frac{0.01541\text{W/m.°C}}{(1.2/12)\text{ ft}}(7.999) = 1.232 \text{ Btu/h.ft}^2\text{.°F}$$
$$A_i = \pi D_i L = \pi(1/12\text{ ft})(50\text{ ft}) = 13.09\text{ ft}^2$$
$$A_o = \pi D_o L = \pi(1.2/12\text{ ft})(50\text{ ft}) = 15.708\text{ ft}^2$$

Using the assumed value of glass temperature, the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_s + T_{surr})(T_s{}^2 + T_{surr}{}^2) \\ &= (0.5)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\text{.R}^4)[(150+460)+(60+460)][(150+460)^2+(60+460)^2]R^3 \\ &= 0.6222 \text{ Btu/ft}^2\text{.R} \end{aligned}$$

Then the combined convection and radiation heat transfer coefficient outside becomes

$$h_{o,combined} = h_o + h_{rad} = 1.232 + 0.6222 = 1.854 \text{ Btu/ft}^2\text{.R}$$

and

$$\dot{Q} = \frac{T_{water} - T_\infty}{\dfrac{1}{h_i A_i} + \dfrac{\ln(D_o/D_i)}{4\pi kL} + \dfrac{1}{h_o A_o}} = \frac{150-60}{\dfrac{1}{(30)(13.09)} + \dfrac{\ln(1.2/1)}{4\pi(30)(50)} + \dfrac{1}{(1.854)(15.708)}} = 2440 \text{ Btu/h}$$

The mass flow rate of water

$$\dot{m} = \rho A_c V = (62.2 \text{ lbm/ft}^3)\ \pi(1/12\text{ ft})^2/4\ (4\text{ ft/s}) = 1.357\text{ lbm/s} = 4885\text{ lbm/h}$$

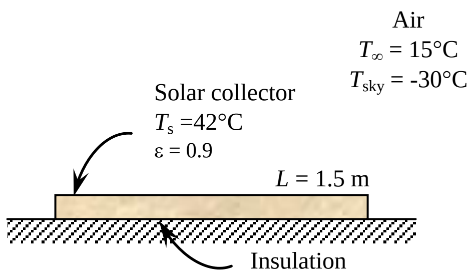
Then the temperature drop of water as it flows through the pipe becomes

$$\dot{Q} = \dot{m}C_p\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{2440 \text{ Btu/h}}{(4885 \text{ lbm/h})(1.0 \text{ Btu/lbm.°F})} = \mathbf{0.50°F}$$

9-112 A flat-plate solar collector placed horizontally on the flat roof of a house is exposed to the calm ambient air. The rate of heat loss from the collector by natural convection and radiation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (42+15)/2 = 28.5°C are (Table A-15)



$$k = 0.02577 \text{ W/m.°C}$$

$$\upsilon = 1.594\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7286$$

$$\beta = \frac{1}{T_f} = \frac{1}{(28.5+273)\text{K}} = 0.003317 \text{K}^{-1}$$

Analysis The characteristic length in this case is determined from

$$L_c = \frac{A_s}{p} = \frac{(1.5\text{ m})(6\text{ m})}{2(1.5\text{ m}+6\text{ m})} = 0.6\text{ m}$$

Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003317\text{K}^{-1})(42-15\text{ K})(0.6\text{ m})^3}{(1.594\times10^{-5}\text{ m}^2/\text{s})^2}(0.7286) = 5.443\times10^8$$

$$Nu = 0.15Ra^{1/3} = 0.15(5.443\times10^8)^{1/3} = 122.5$$

$$h = \frac{k}{L_c}Nu = \frac{0.02577\text{ W/m.°C}}{0.6\text{ m}}(122.5) = 5.26\text{ W/m}^2.\text{°C}$$

$$A_s = (1.5\text{ m})(6\text{ m}) = 9\text{ m}^2$$

and

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (5.26\text{ W/m}^2.\text{°C})(9\text{ m}^2)(42-15)\text{°C} = \mathbf{1278\ W}$$

Heat transfer rate by radiation is

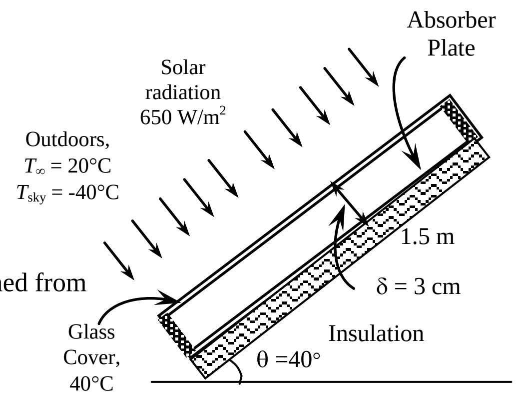
$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)$$
$$= (0.9)(9\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(42+273\text{K})^4 - (-30+273\text{K})^4] = \mathbf{2920\ W}$$

9-113 A flat-plate solar collector tilted 40°C from the horizontal is exposed to the calm ambient air. The total rate of heat loss from the collector, the collector efficiency, and the temperature rise of water in the collector are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** There is no heat loss from the back surface of the absorber plate.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+20)/2 = 30$°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\upsilon = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7282$$
$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033 \text{K}^{-1}$$



Analysis (*a*) The characteristic length in this case is determined from

$$L_c = \frac{A_s}{p} = \frac{(1.5\text{ m})(2\text{ m})}{2(1.5\text{ m}+2\text{ m})} = 0.429 \text{ m}^2$$

Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(\cos 40°)(0.003311\text{ K}^{-1})(40-20\text{ K})(0.429\text{ m})^3}{(1.608\times10^{-5}\text{ m}^2\text{/s})^2}(0.7282) = 1.100\times10^8$$

$$Nu = 0.15Ra^{1/3} = 0.15(1.100\times10^8)^{1/3} = 71.87$$
$$h = \frac{k}{L_s}Nu = \frac{0.02588\text{ W/m.°C}}{0.429\text{ m}}(71.87) = 4.340\text{ W/m}^2\text{.°C}$$
$$A_s = (1.5\text{ m})(2\text{ m}) = 3\text{ m}^2$$

and

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (4.340\text{ W/m}^2\text{.°C})(3\text{ m}^2)(40-20)\text{°C} = 260.4\text{ W}$$

Heat transfer rate by radiation is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma({T_{surr}}^4 - {T_s}^4)$$
$$= (0.9)(3\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(40+273\text{K})^4 - (-40+273\text{K})^4] = 1018\text{W}$$

and

$$\dot{Q}_{total} = 260.4 + 1018 = \mathbf{1279\text{ W}}$$

(*b*) The solar energy incident on the collector is

$$\dot{Q}_{incident} = \alpha\dot{q}A_s = (0.88)(650\text{ W/m}^2)(3\text{ m}^2) = 1716\text{ W}$$

Then the collector efficiency becomes

$$efficiency = \frac{\dot{Q}_{incident} - \dot{Q}_{lost}}{\dot{Q}_{incident}} = \frac{1716-1279}{1716} = 0.255 = \mathbf{25.5\%}$$

(*c*) The temperature rise of the water as it passes through the collector is

$$\dot{Q} = \dot{m}C_p\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}C_p} = \frac{(1716-1279)\text{W}}{(1/60\text{ kg/s})(4180\text{ J/kg.°C})} = \mathbf{6.3\text{ °C}}$$

9-114 ….. 9-117 Design and Essay Problems